# The weak interaction
# Part II


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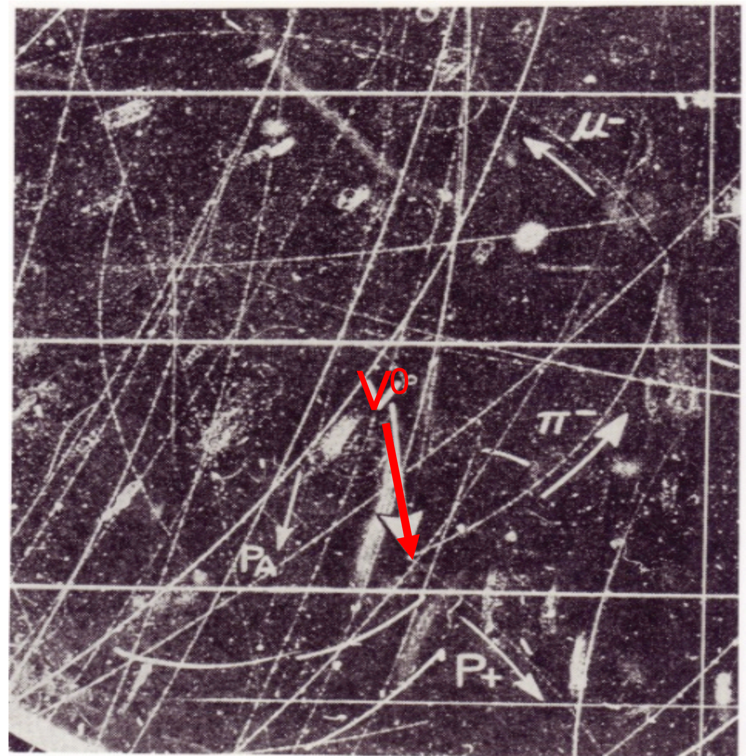
- The $K^0$-$\bar{K}^0$ system
- The CKM mechanism
- Measurements of the unitarity triangle parameters : some examples

# The $K^0$-$\bar{K}^0$ system

Remember the strange particles ?

**Observation of Long-Lived Neutral *V* Particles***

K. Lande, E. T. Booth, J. Impeduglia, and L. M. Lederman,
*Columbia University, New York, New York*
and
W. Chinowsky, *Brookhaven National Laboratory, Upton, New York*
(Received July 30, 1956)

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Brookhaven, 1956

Two states :

$|K_1\rangle \rightarrow \pi\pi$

$|K_2\rangle \rightarrow \pi\pi\pi$

Same mass (~ 500 MeV)

Very different lifetimes

M(π) ~ 140 MeV
M(K) ~ 500 MeV

$K_2$ lifetime ~ 10000 $K_1$ lifetime due to phase space

# CP violation in the $K^0$ system

*NB : $K_1 = K_S$ and $K_2 = K_L$*

$|K^0\rangle = |s\bar{d}\rangle$

$|\bar{K}^0\rangle = |\bar{d}s\rangle$

$CP|K^0\rangle = |\bar{K}^0\rangle \Rightarrow$ $|K^0\rangle = |s\bar{d}\rangle$, $|\bar{K}^0\rangle = |\bar{d}s\rangle$ — not CP eigenstates

One can build :

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

CP eigenstates

$|K_1\rangle \rightarrow \pi\pi$ $|K_2\rangle \rightarrow \pi\pi\pi$

CP(ππ) = +1 and CP(πππ) = -1

if CP is a good symmetry for the weak interaction : ~~$|K_2\rangle \rightarrow \pi\pi$~~

$|K_1\rangle \rightarrow \pi\pi$

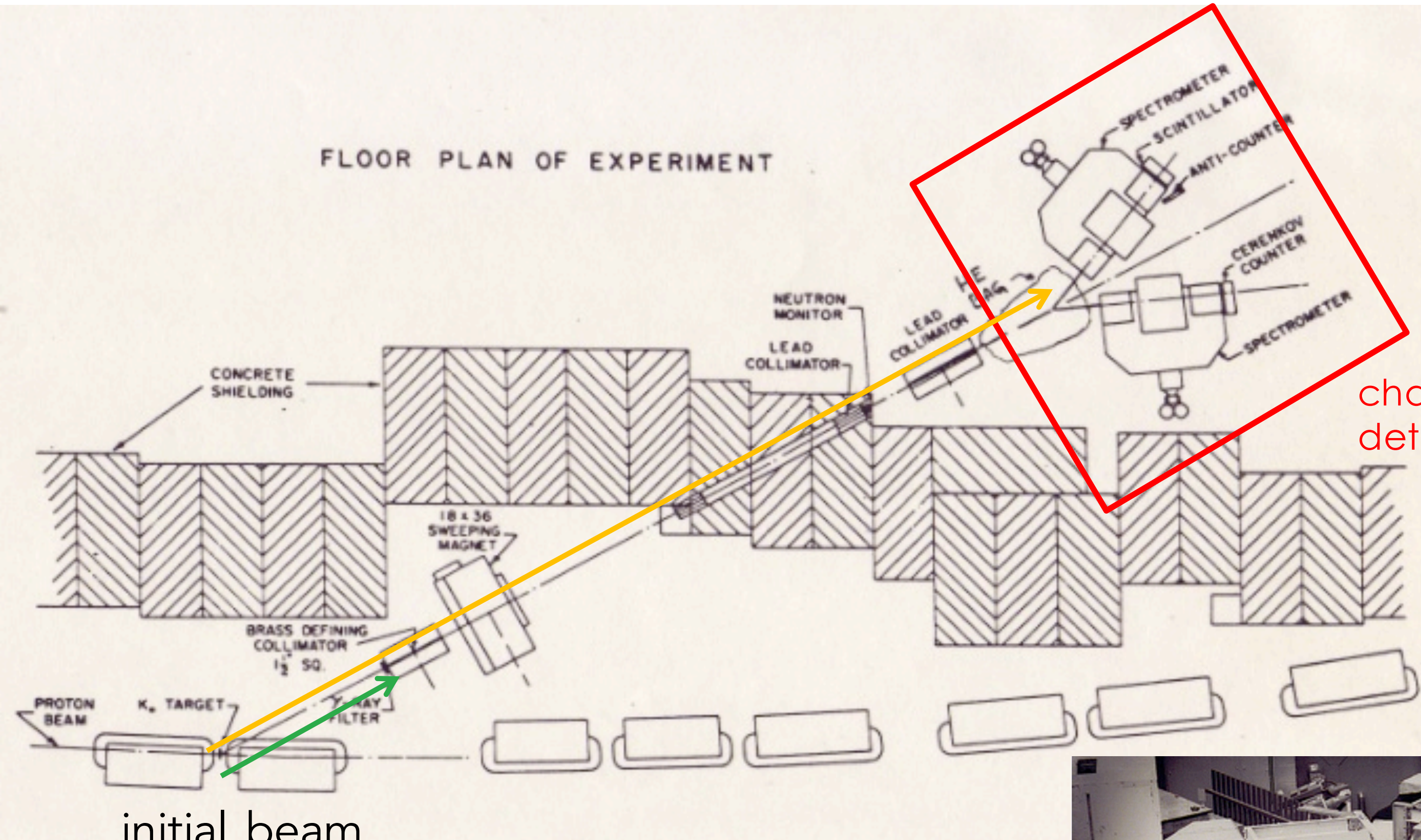
After some time, pure $K_2$ beam

initial beam
$K_1$ and $K_2$

Search for the signal of the decay $|K_2\rangle \rightarrow \pi\pi$ far (20 meters) from the production point of the $K_1$ and $K_2$

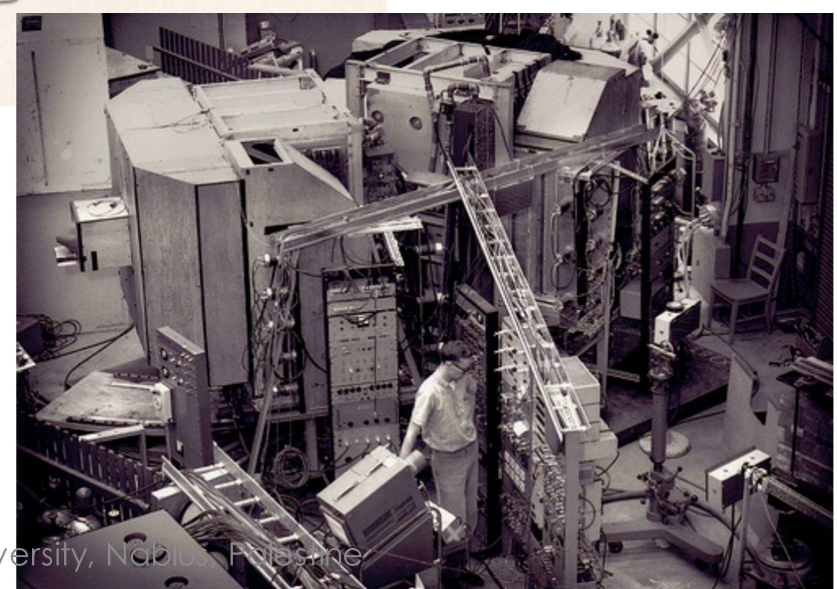
?

# Cronin& Fitch experiment 1964




charged tracks detector

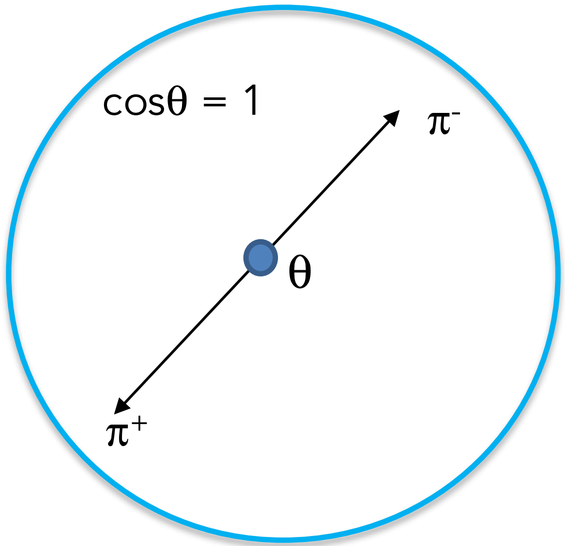
initial beam
$K_1$ and $K_2$



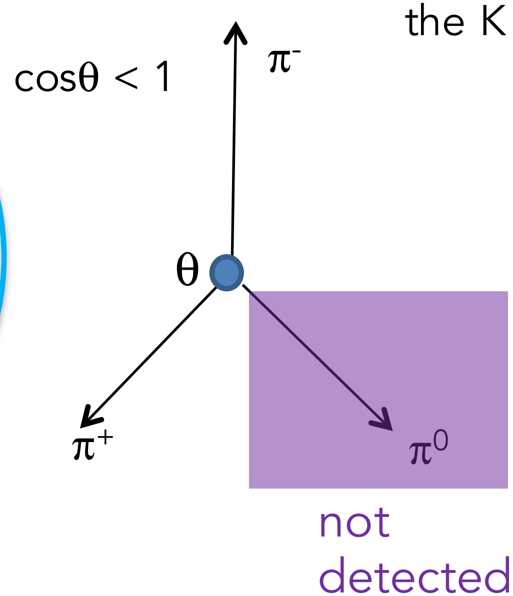
Two informations :

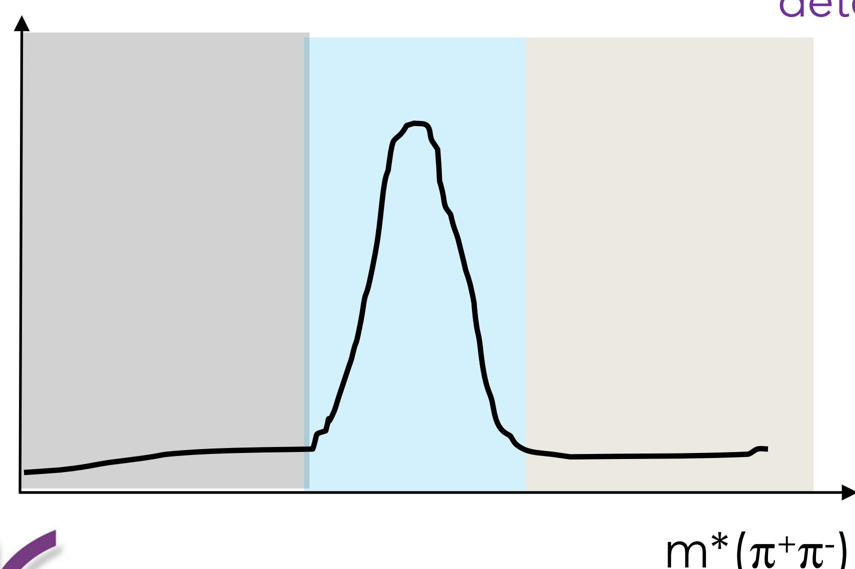
- The $\pi^+\pi^-$ invariant mass (m*)
- The opening angle between the two pions in the K center of mass frame

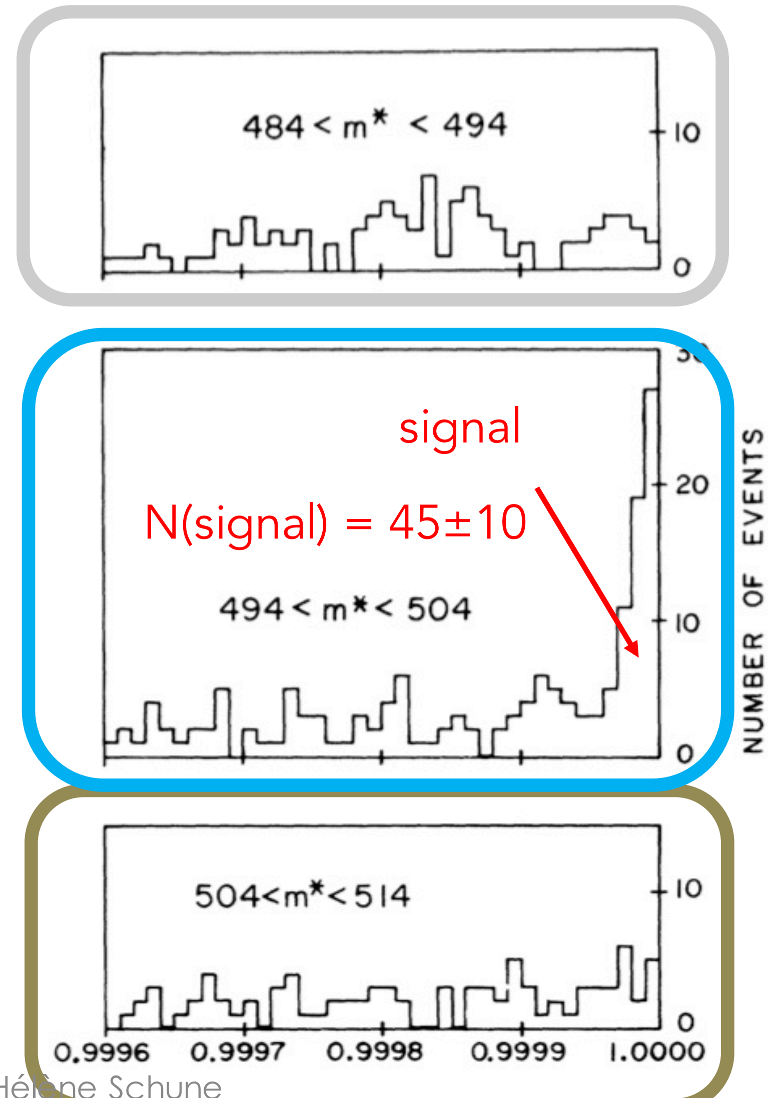
signal




background



VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

1964

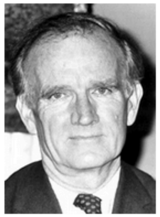
EVIDENCE FOR THE $2\pi$ DECAY OF THE $K_2^0$ MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

We would conclude therefore that $K_2^0$ decays to two pions with a branching ratio $R = (K_2 \to \pi^+ + \pi^-)/(K_2^0 \to \text{all charged modes}) = (2.0 \pm 0.4) \times 10^{-3}$ where the error is the standard deviation. As emphasized above, any alternate explanation of the effect requires highly nonphysical behavior of the three-body decays of the $K_2^0$. The presence of a two-pion decay mode implies that the $K_2^0$ meson is not a pure eigenstate of $CP$. Expressed as

## The Nobel Prize in Physics 1980

James Watson Cronin
Prize share: 1/2



Val Logsdon Fitch
Prize share: 1/2

The Nobel Prize in Physics 1980 was awarded jointly to James Watson Cronin and Val Logsdon Fitch *"for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"*

R. Turlay was a PhD student
J Christenson was a graduate student

« The discovery emphasizes, once again, that even almost self evident principles in science cannot be regarded fully valid until they have been critically examined in precise experiments. »

Today :

$$\frac{A\left(|K_2\rangle \to \pi\pi\right)}{A\left(|K_1\rangle \to \pi\pi\right)} = (2.271 \pm 0.017)10^{-3}$$

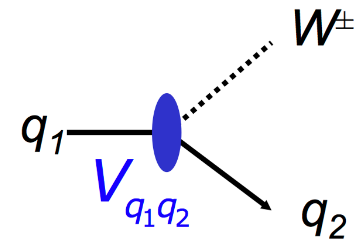
0.7 % precision !

Experimental observation of CP violation in K decays

+ Cabibbo angle

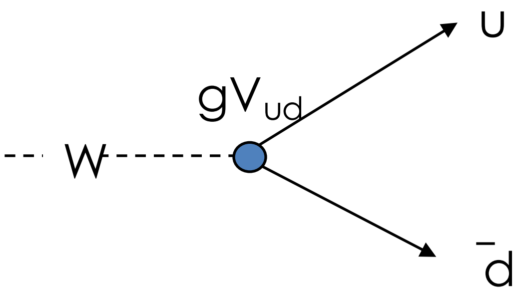
$V_{CKM}$ Cabibbo-Kobayashi-Maskawa matrix

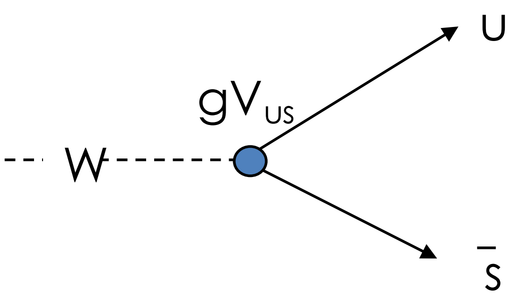
# $V_{CKM}$ Cabibbo-Kobayashi-Maskawa matrix

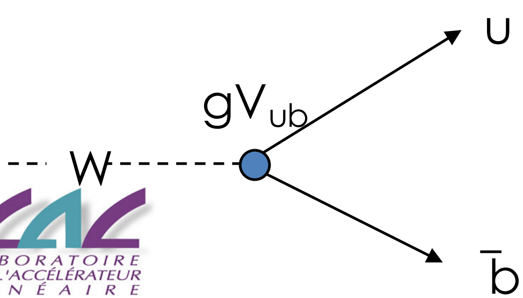



## Two different way of seeing the charged interactions among quarks

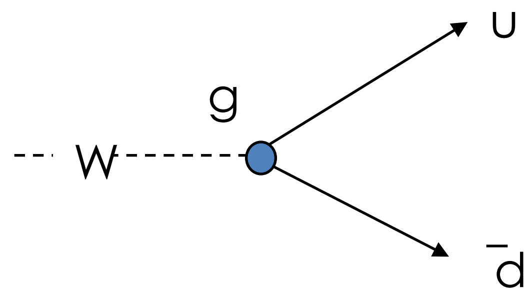
In the basis dealing with mass eigenstates :




In the basis where : charged interactions are just between members of the same family and « CKM » is diagonal




Weak interaction eigenstates $\neq$ Mass eigenstates (flavour or strong interaction eigenstates)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

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Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

# *CP*-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

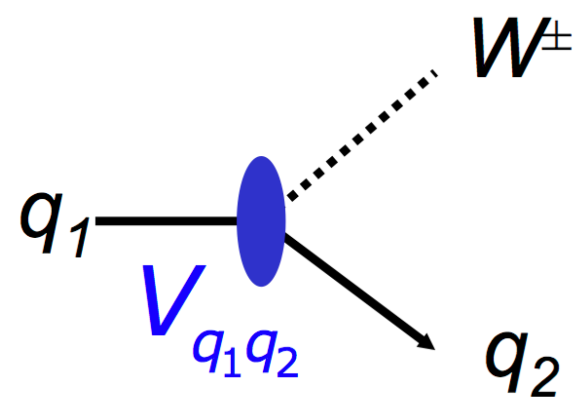
*Department of Physics, Kyoto University, Kyoto*

1973
Before the discovery of the $4^{th}$ quark

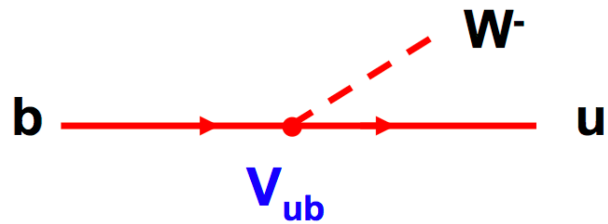
Prediction of the $3^{rd}$ family

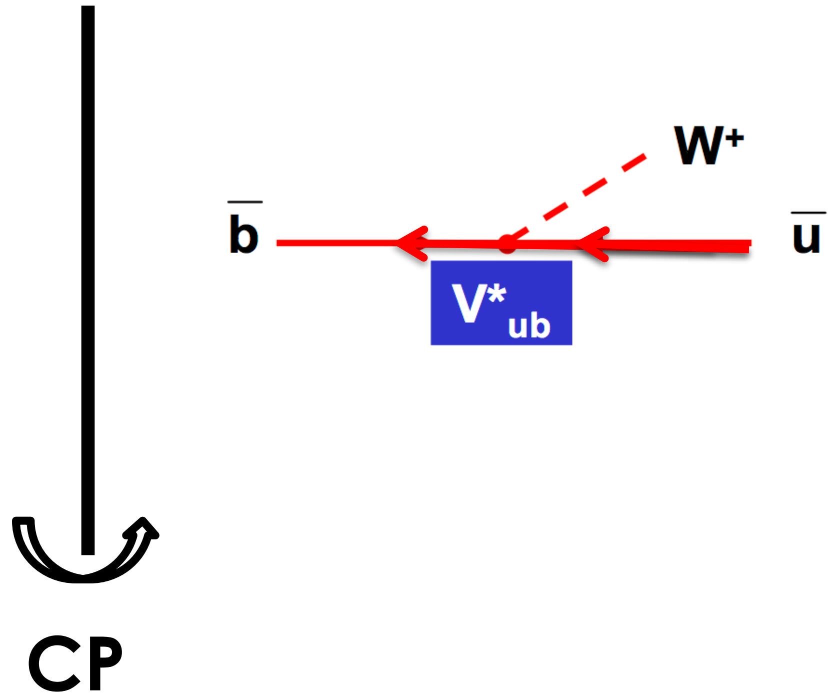
| # families | # angles | # reducible phases | # irreducible phases |
|---|---|---|---|
| n | n(n-1)/2 | 2n-1 | n(n+1)/2 –(2n-1)=(n-1)(n-2)/2 |
| 2 | 1 | | 0 |
| 3 | 3 | | 1 |
| 4 | 6 | | 3 |

$$\begin{pmatrix} u & c & t \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



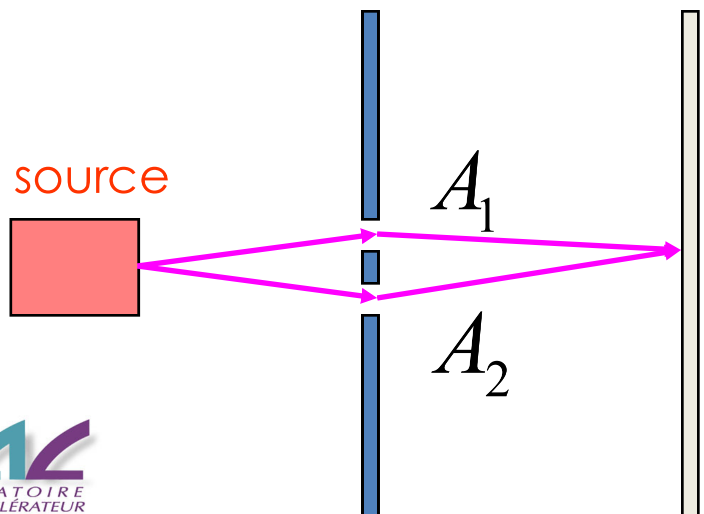

$V_{CKM}$ Cabibbo-Kobayashi-Maskawa matrix

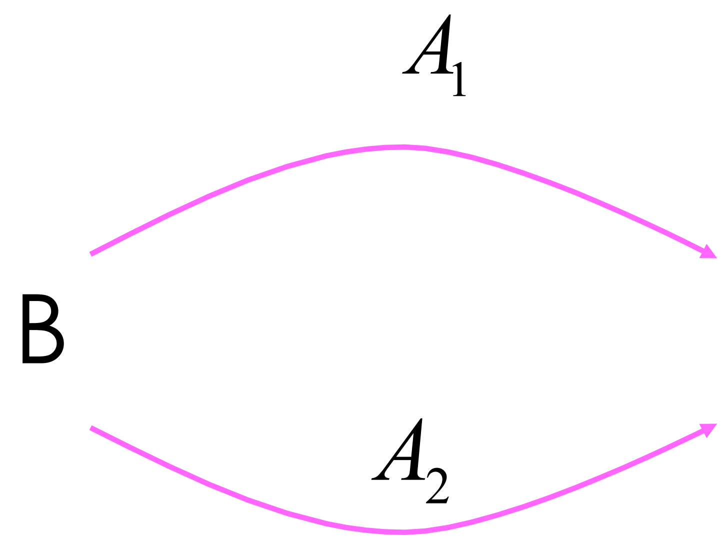



$V^*_{ub} \neq V_{ub} \rightarrow$ CP violation

One amplitude : no sensitivity on phase ($|V_{ij}|^2 = |V^*_{ij}|^2$ )

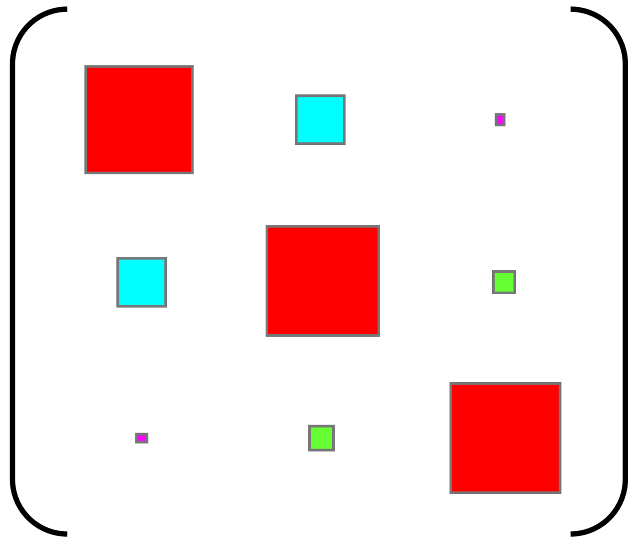


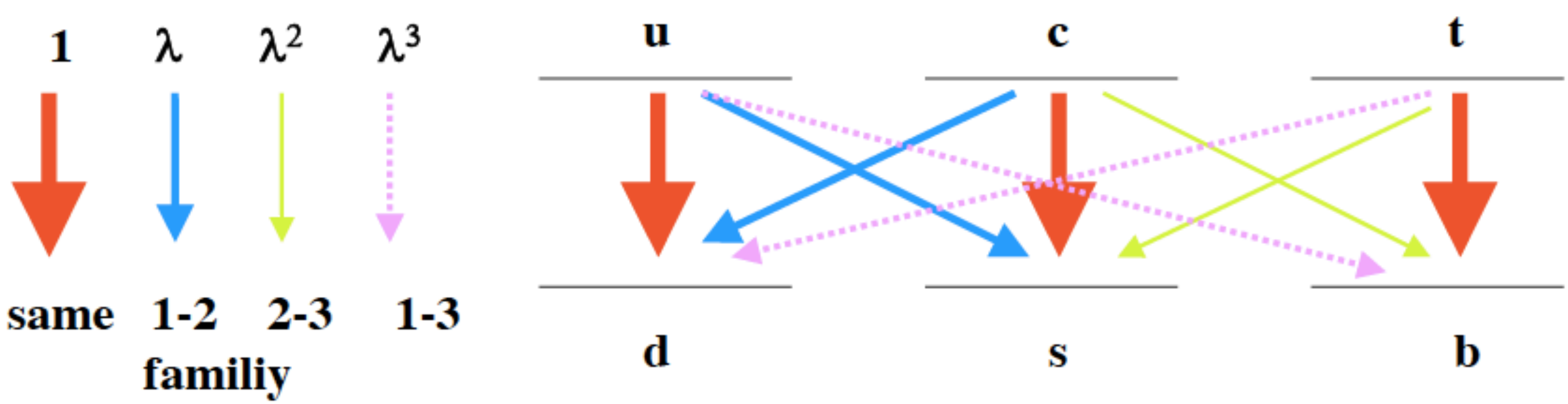



No prediction on the $V_{ij}$ → they need to be measured

→ Experimental observations :

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$





3 families ⎱ 3 angles ($\theta_{ij}$) and one phase ($\delta$)

$V_{ub}$

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij}$$
$$s_{ij} = \sin\theta_{ij}$$

→ Parametrization in power of $\lambda$ ($=\sin\vartheta_c$) $= s_{12} = |V_{us}| \sim 0.22$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \vartheta(\lambda^4)$$

$$\lambda = \sin\theta_c \sim 0.22$$
$$A \sim 0.80$$
$$\rho \sim 0.20$$
$$\eta \sim 0.35$$

# Measuring triangles

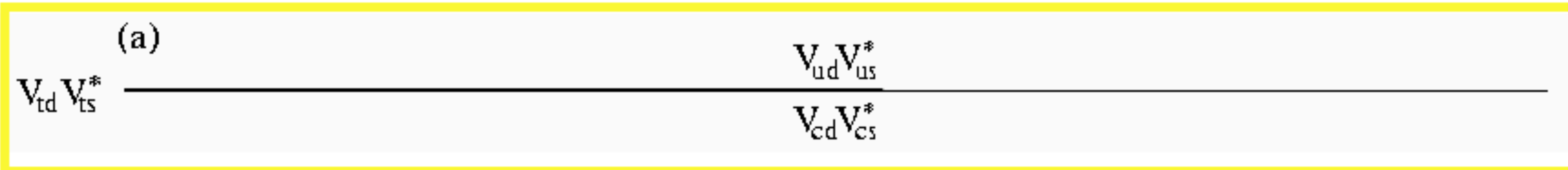
*Stay within the 3 families*

$$\begin{pmatrix} u & c & t \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Unitarity of $V_{CKM}$ $\quad VV^{\dagger} = V^{\dagger}V = 1$

→9 relations $\quad \sum_{k=1}^{n} V_{ik} V_{jk}^{*} = \delta_{ij},$

The non-diagonal elements of the matrix products correspond to 6 triangle equations




$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0 \qquad \lambda \; \lambda \; \lambda^5$$

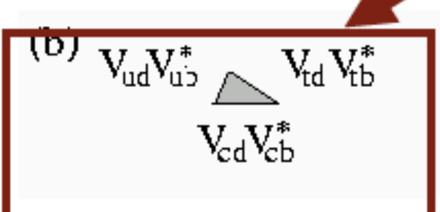
$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \qquad \lambda^3 \; \lambda^3 \; \lambda^3$$

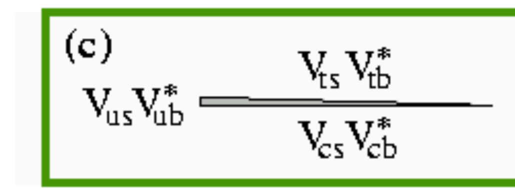
$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0 \qquad \lambda^4 \; \lambda^2 \; \lambda^2$$

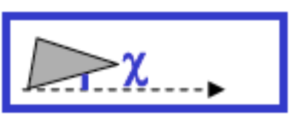
$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0 \qquad \lambda^3 \; \lambda^3 \; \lambda^3$$

$$V_{td}^* V_{cd} + V_{ts}^* V_{cs} + V_{tb}^* V_{cb} = 0 \qquad \lambda^4 \; \lambda^2 \; \lambda^2$$

$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0 \qquad \lambda \; \lambda \; \lambda^5$$




They all have the same area, proportionnal to the amount of CP violation in the SM

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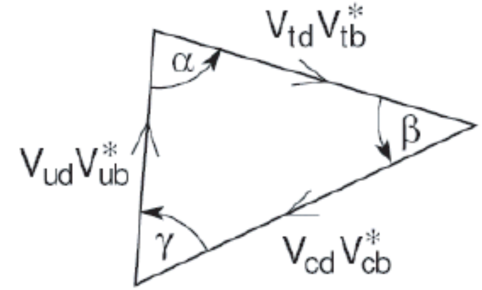
"the" unitarity triangle : $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$



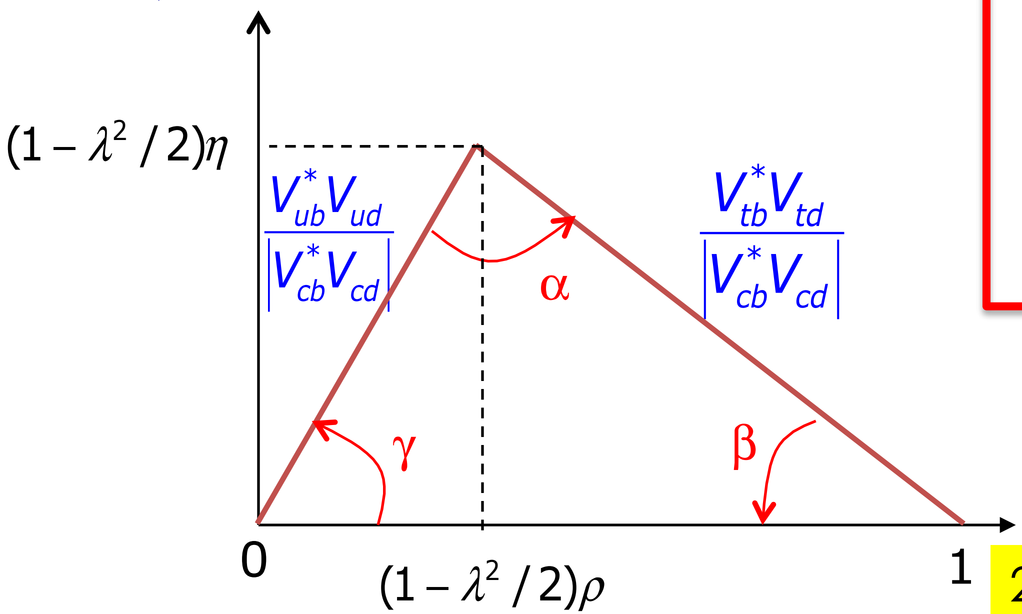

$$V_{td}\,V_{tb}^* = A\lambda^3(1-\rho-i\eta) + A\lambda^5(\rho+i\eta)$$

$$V_{ud}\,V_{ub}^* = A\lambda^3(\rho+i\eta)\times\left(1-\frac{\lambda^2}{2}\right)$$

at order $\lambda^5$

$$V_{cd}\,V_{cb}^* = -\,A\lambda^3$$

Basis of the triangle aligned on the real axis, normalized to 1




$$\beta = \arg\left(\frac{V_{td}V_{tb}^*}{|V_{cd}V_{cb}^*|}\right) = \mathrm{atan}\left(\frac{(1-\lambda^2/2)\eta}{1-(1-\lambda^2/2)\rho}\right)$$

$$\gamma = \arg\left(\frac{V_{ud}V_{ub}^*}{|V_{cd}V_{cb}^*|}\right) = \mathrm{atan}\left(\frac{\eta}{\rho}\right)$$

$$\alpha+\beta+\gamma = \pi$$

2 sides ; 3 angles

⇒ aim : to overconstrain this unitarity triangle

precision test of the Standard **Model**



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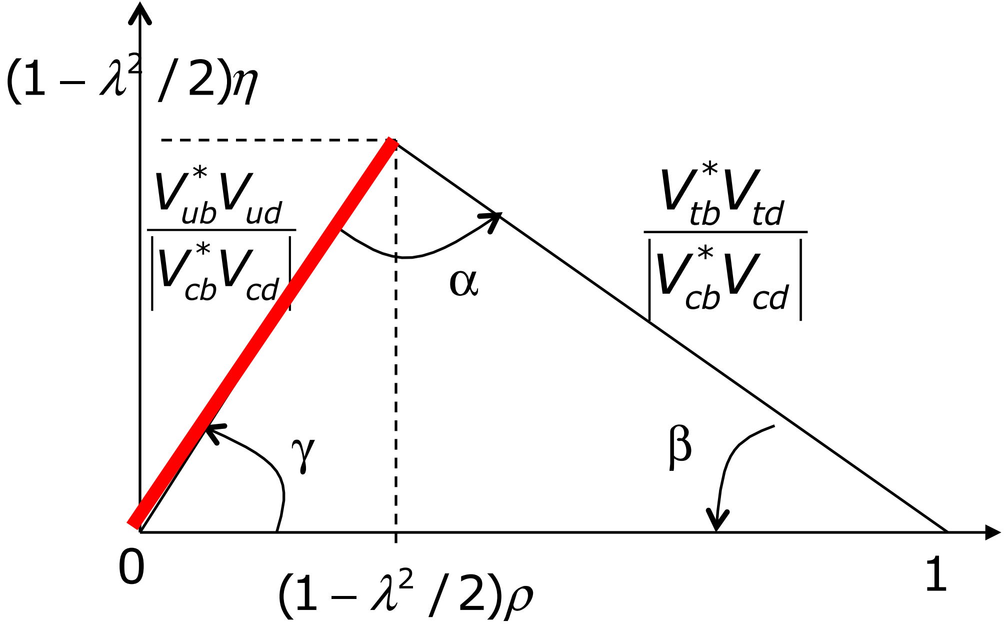
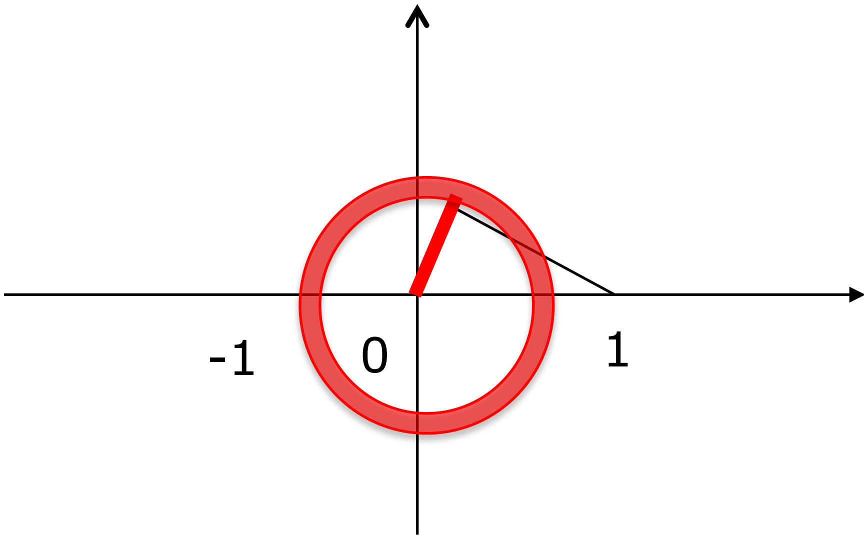
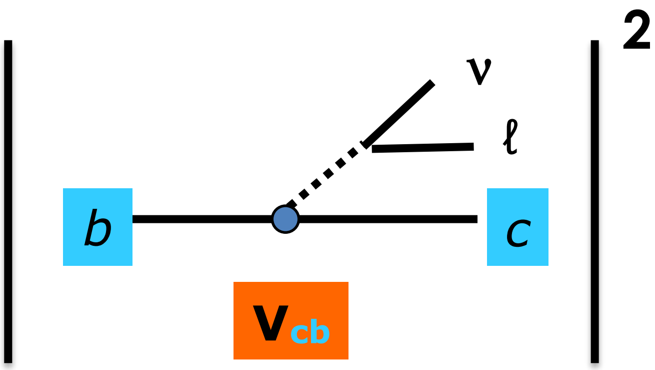
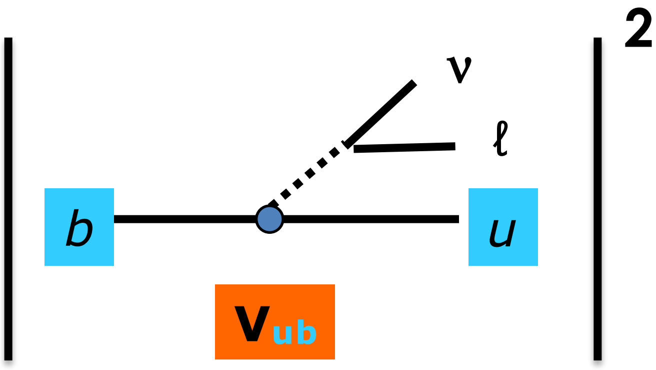
# Measurements of the unitarity triangle parameters : some examples



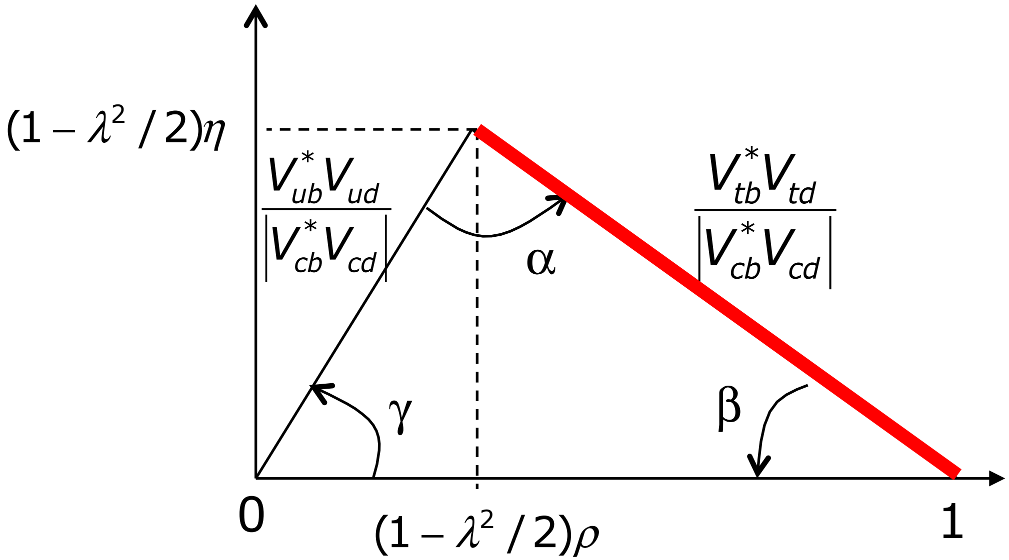

Rates of semileptonic B decays

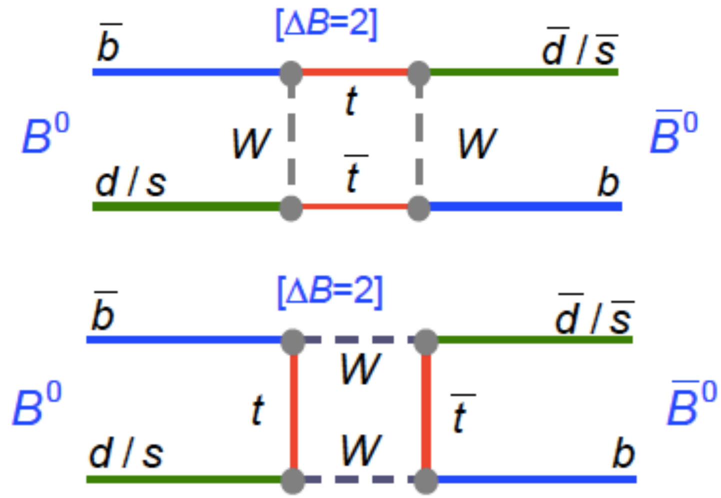



Conceptually simple, complicated by QCD

# The other side : $B^0$-$\overline{B}^0$ oscillations




Diagrams involving $V_{td}$ or $V_{ts}$

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The mixing phenomenon

Pairs of self-conjugate mesons that can be transformed to each other via flavour changing weak interaction transitions are:

$$|K^0\rangle = |\bar{s}d\rangle \qquad |D^0\rangle = |c\bar{u}\rangle \qquad |B_d^0\rangle = |\bar{b}d\rangle \qquad |B_s^0\rangle = |\bar{b}s\rangle$$

They are **flavour eigenstates** with definite quark content $|B^0\rangle, \ |\bar{B}^0\rangle$

- useful to understand particle production and decay

Apart from the flavour eigenstates there are **mass eigenstates**: $|B_L\rangle, \ |B_H\rangle$

- eigenstates of the Hamiltonian
- states of definite mass and lifetime
- They are propagating through space-time

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

Since flavour eigenstates are not mass eigenstates, the flavour eigenstates are mixed with one another as they propagate through space and time


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$$|B_{H,L}(t)\rangle = e^{-i\left(M_{H,L} - i\frac{\Gamma_{H,L}}{2}\right)t} |B_{H,L}(t=0)\rangle$$

+

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$$

Time evolution

The probability to observe a $B^0$ at time t if a $B^0$ was produced at time t=0 is :

$$\left|\langle B^0|H|B^0(t)\rangle\right|^2 = \frac{e^{-\Gamma t}}{2}(1+\cos\Delta m t)$$

The probability to observe a $B^0$ at time t if a $\overline{B^0}$ was produced at time t=0 is :

$$\left|\langle \overline{B}^0|H|B^0(t)\rangle\right|^2 = \frac{e^{-\Gamma t}}{2}(1-\cos\Delta m t)$$

This is the mixing phenomenon !

$$\frac{N_{Unmixed} - N_{Mixed}}{N_{Unmixed} + N_{Mixed}} \sim \cos\Delta m t$$

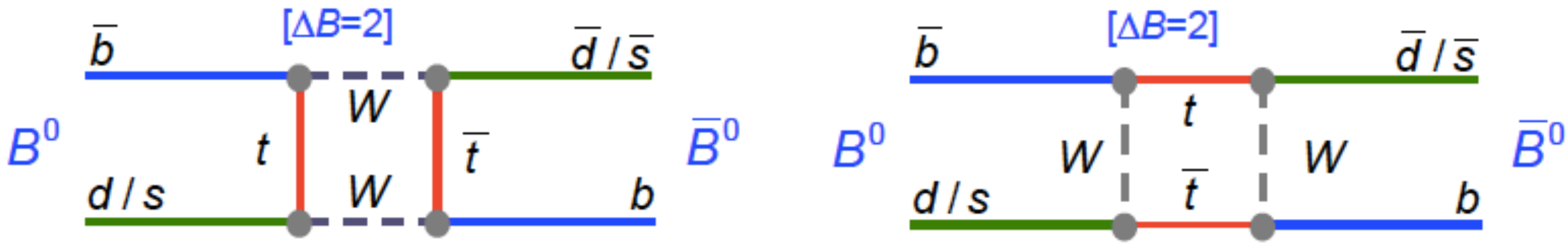
Simplified formulae assuming that the two mass eigenstates have the same lifetime and neglecting CP violation (q/p=1)

)

Let's come back to the unitarity triangle

# Δm can be computed in the Standard Model

Effective FCNC Processes (*CP* conserving — top loop dominates in box diagram):




Perturbative QCD

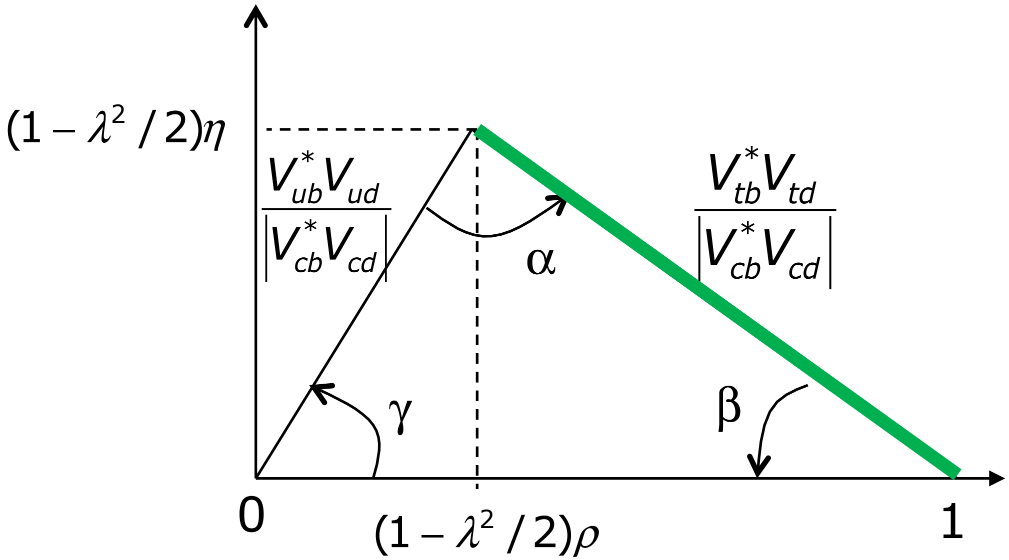
CKM Matrix Elements

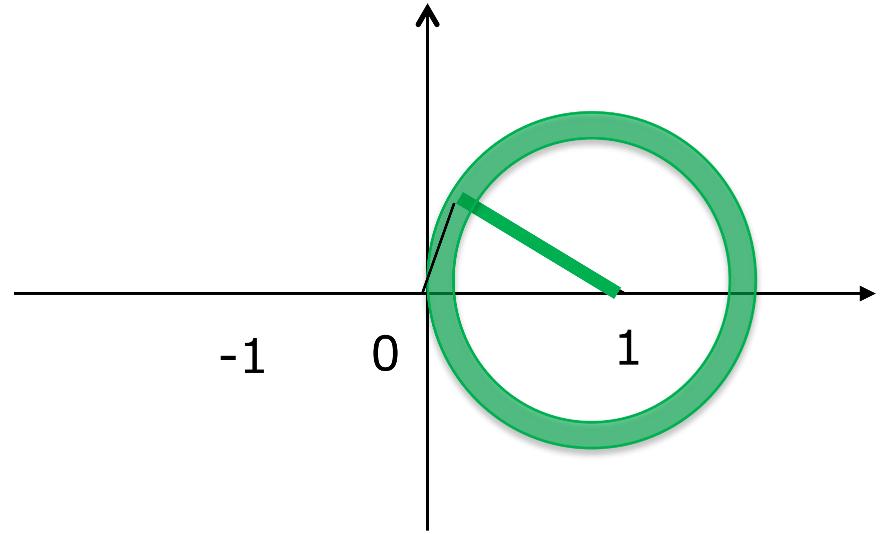
$$\Delta m_q = \frac{G_F^2}{6\pi^2} m_{B_q} m_W^2 \, \eta_B \, S(x_t) \, f_{B_q}^2 B_q \left| V_{tq} V_{tb}^* \right|^2 \quad (\text{for } q = d, s)$$

Loop integral
(top loop dominates)

Non-perturbative QCD :
dominant theoretical
uncertainty

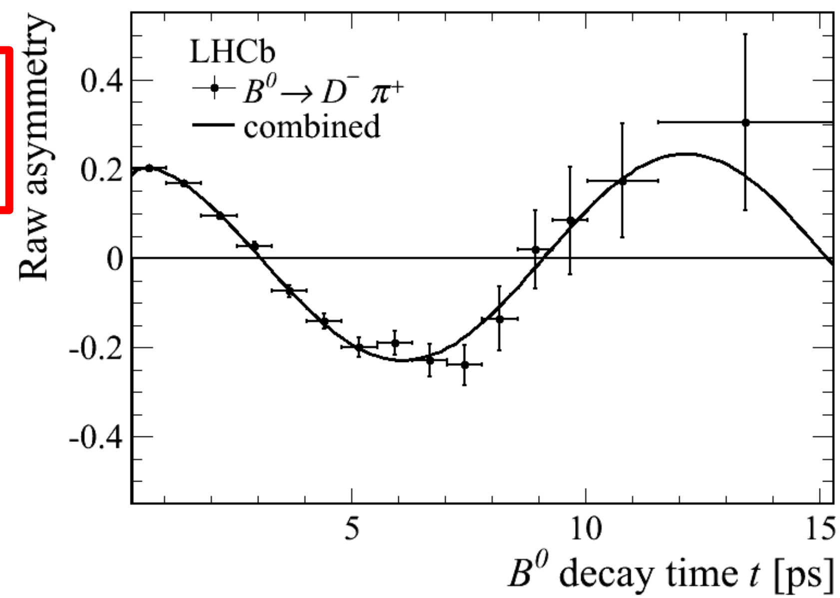
# The other side : $B^0$-$\bar{B}^0$ oscillations

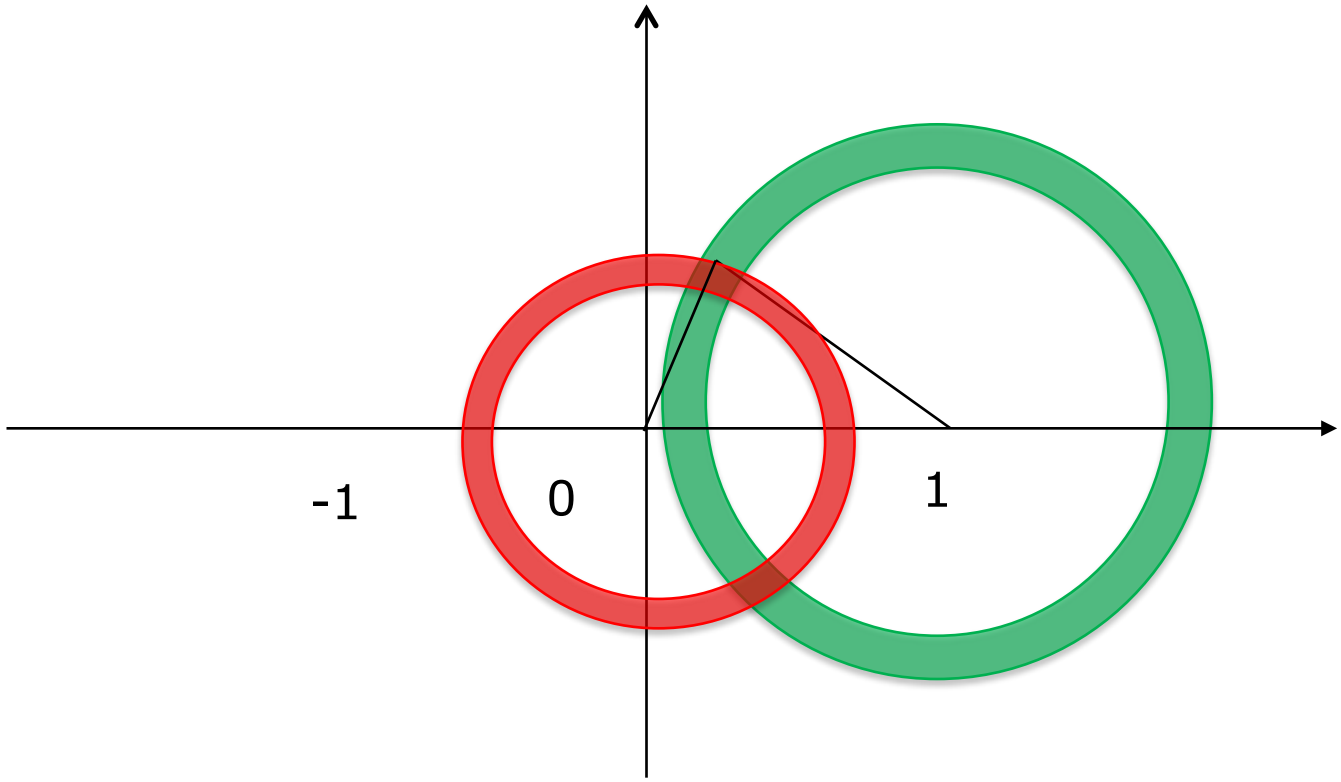



$$\frac{N_{Unmixed} - N_{Mixed}}{N_{Unmixed} + N_{Mixed}} \sim \cos \Delta m t$$

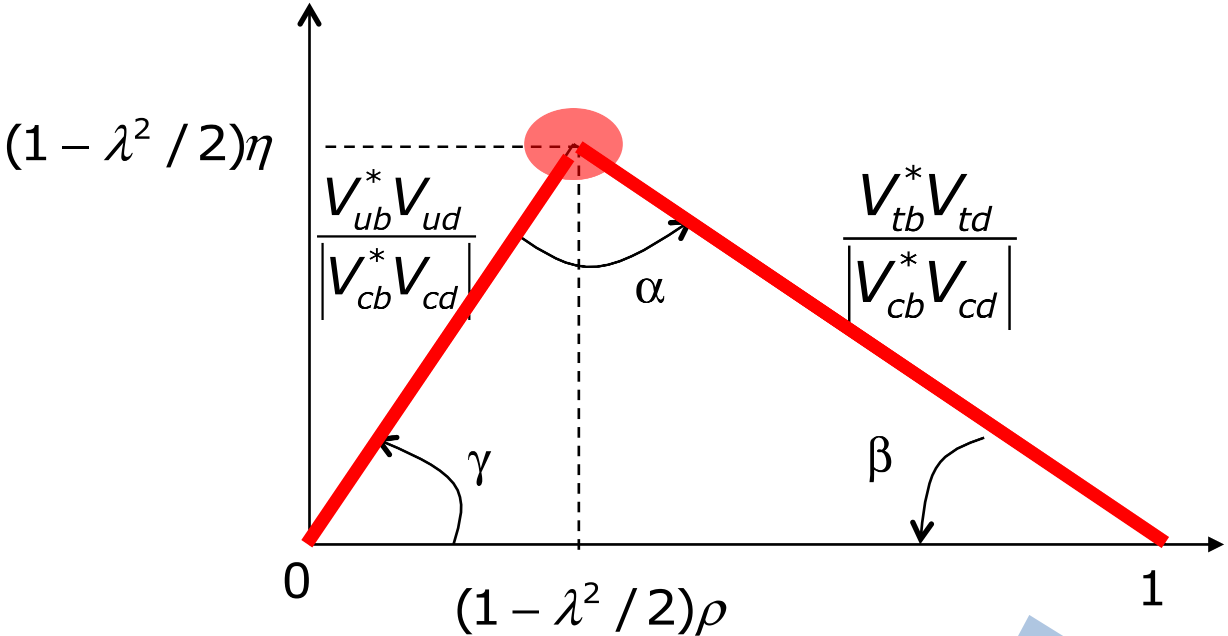
$$\Delta m_d \propto |V_{td}V^*_{tb}|^2$$



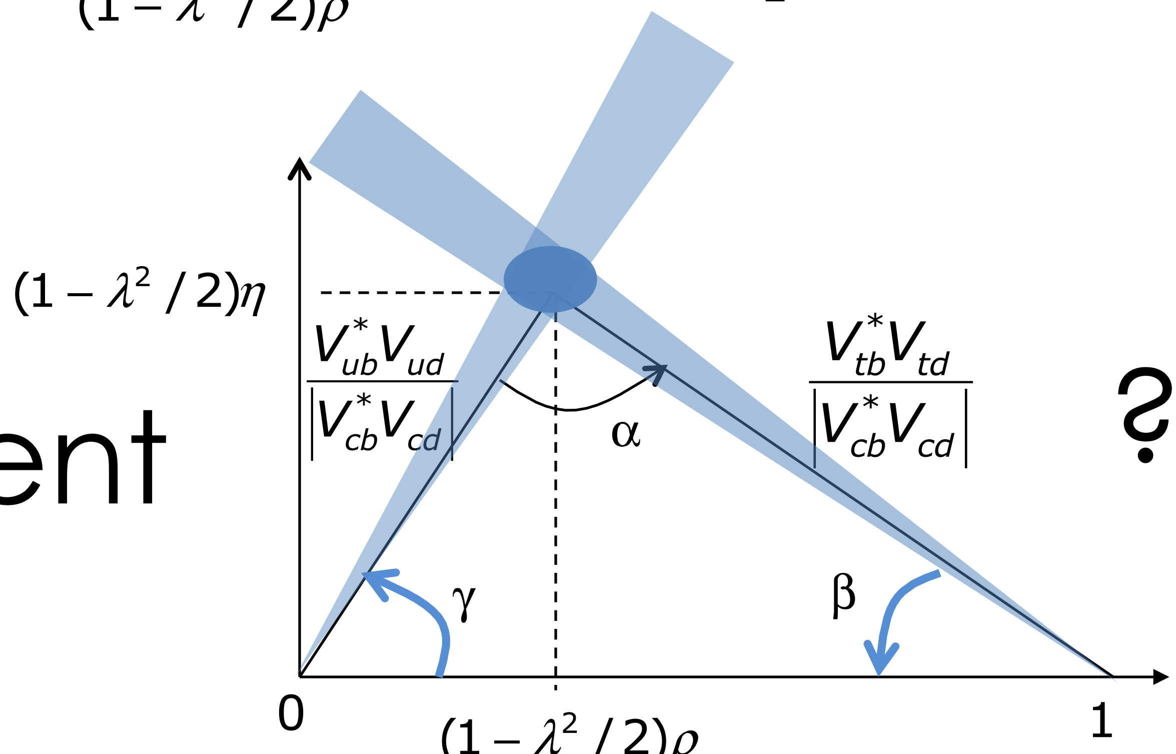



Are the two types of measurements compatible ?

Is




in agreement with

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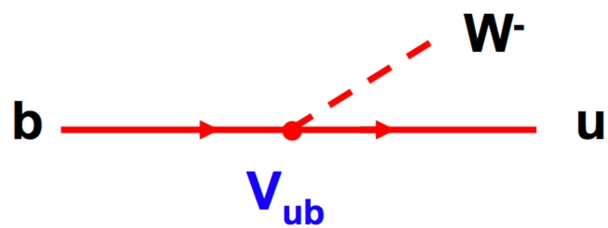
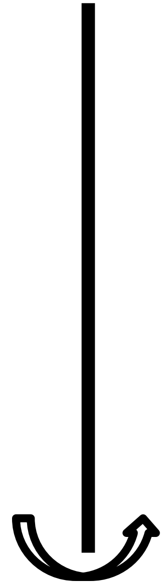
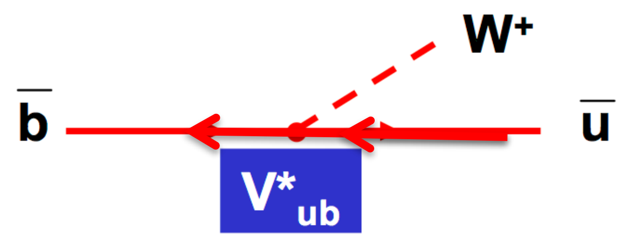
CP violation



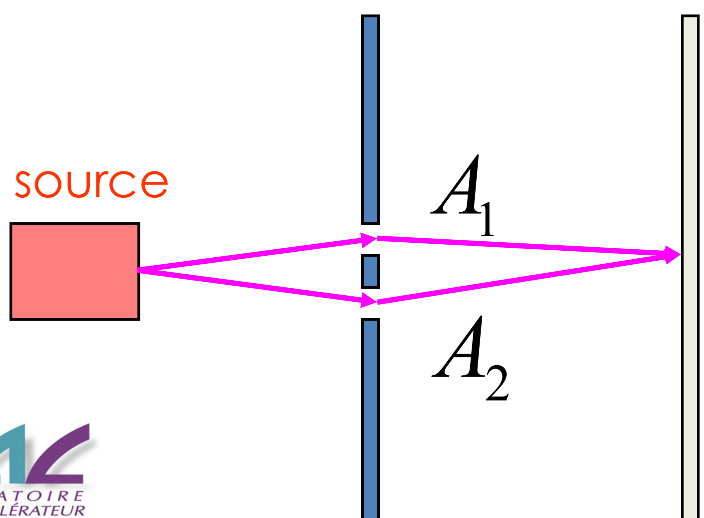



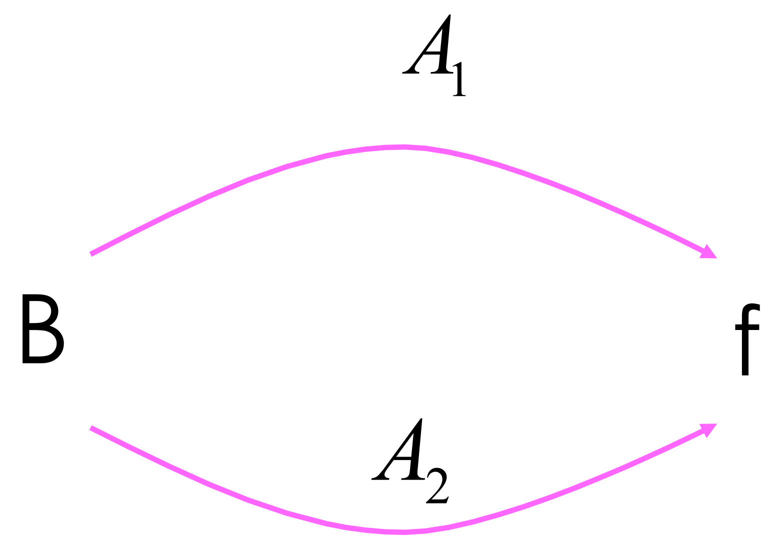



$V^*_{ub} \neq V_{ub}$ → CP violation

**CP**

If you just have one amplitude : no sensitivity on phase ( $|V_{ij}|^2 = |V^*_{ij}|^2$ )



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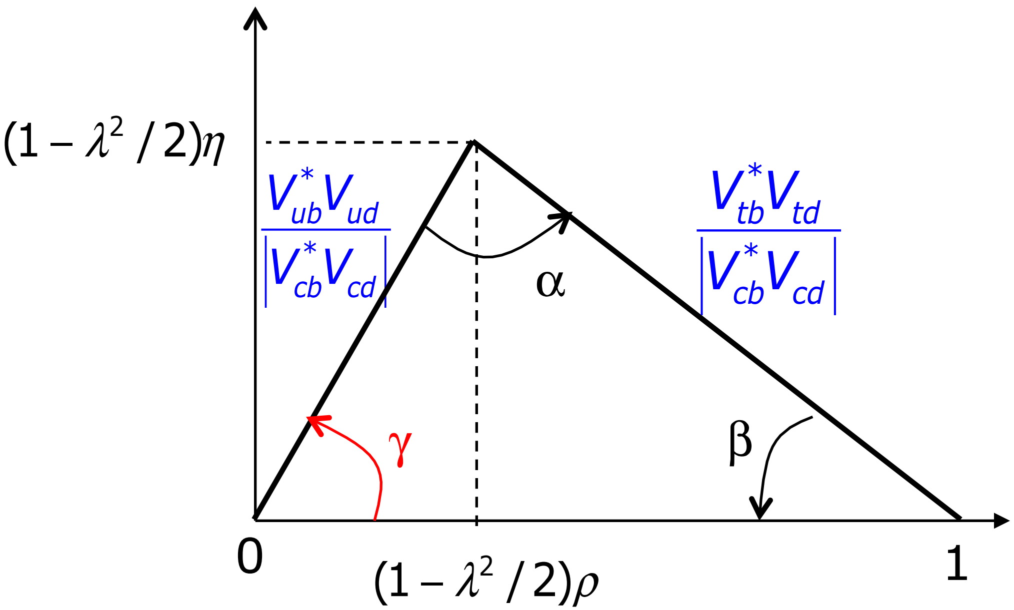
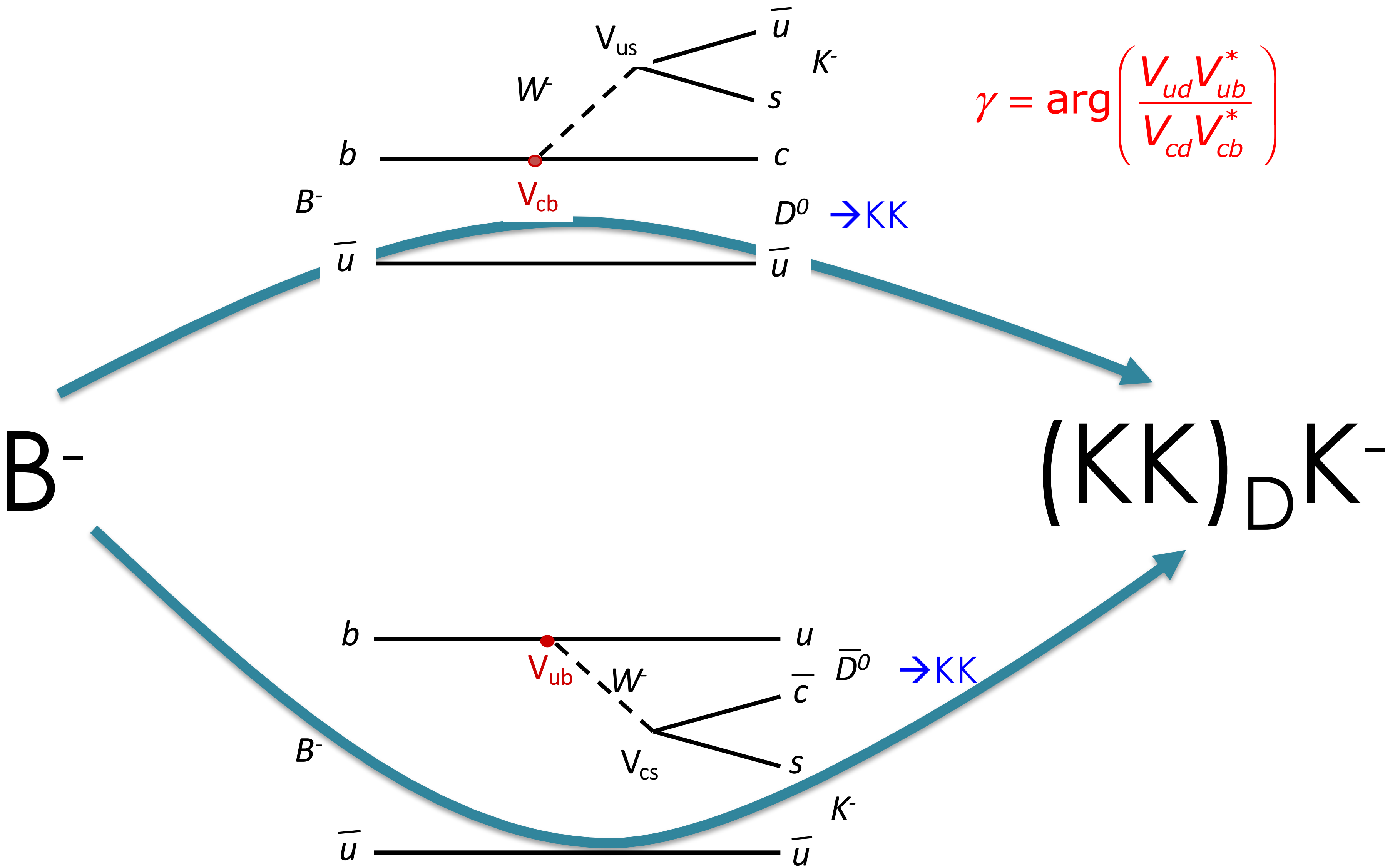
Let's come back to the unitarity triangle “the” unitarity triangle : $V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0$

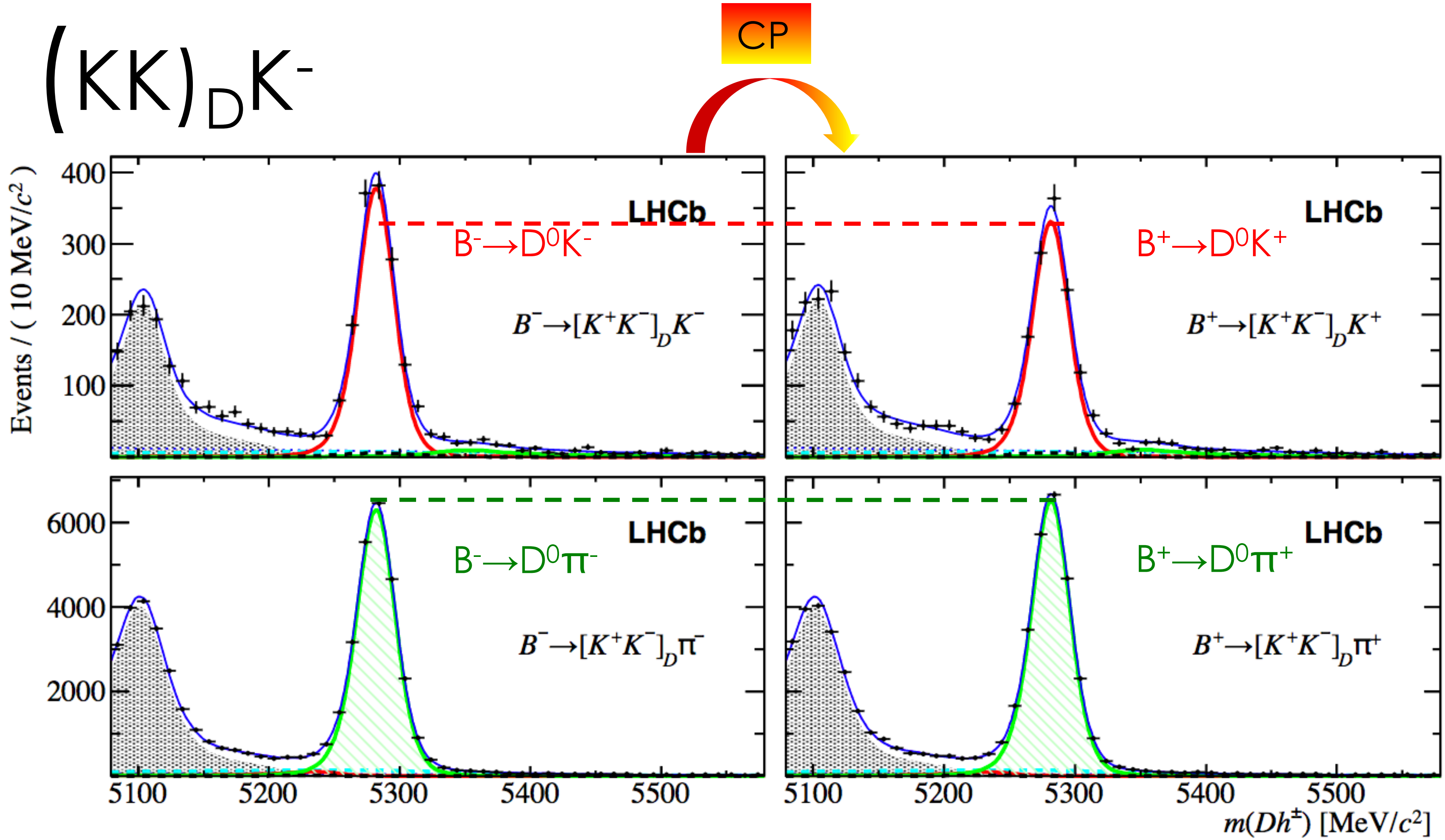



$$\gamma = \arg\left(\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right)$$




$$\gamma = \arg\left(\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

# $(KK)_D K^-$



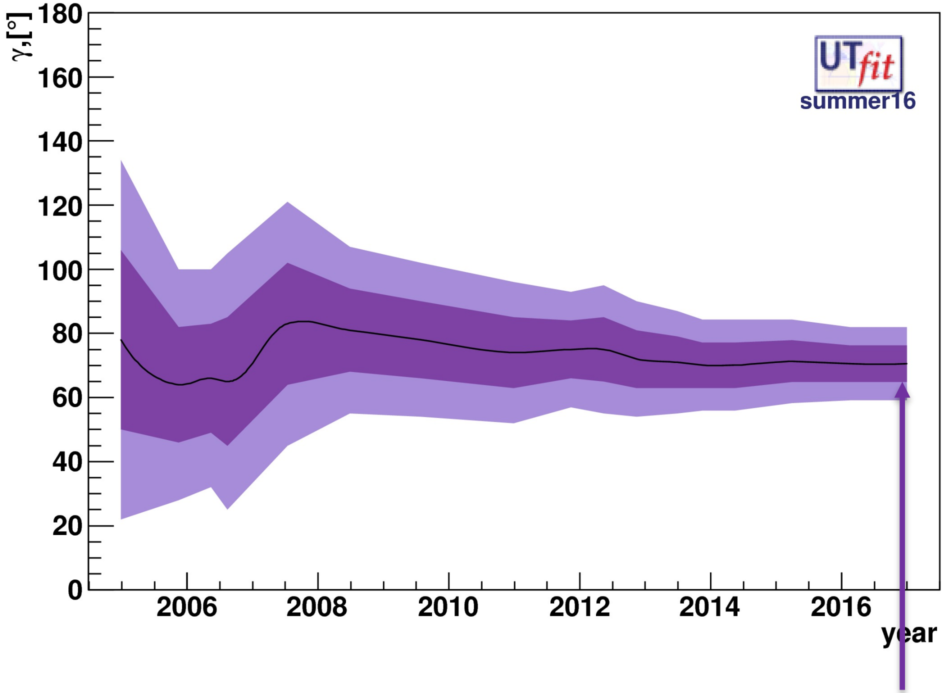

$$A_K^{KK} = 0.087 \pm 0.020 \pm 0.008$$

significantly different from 0 !

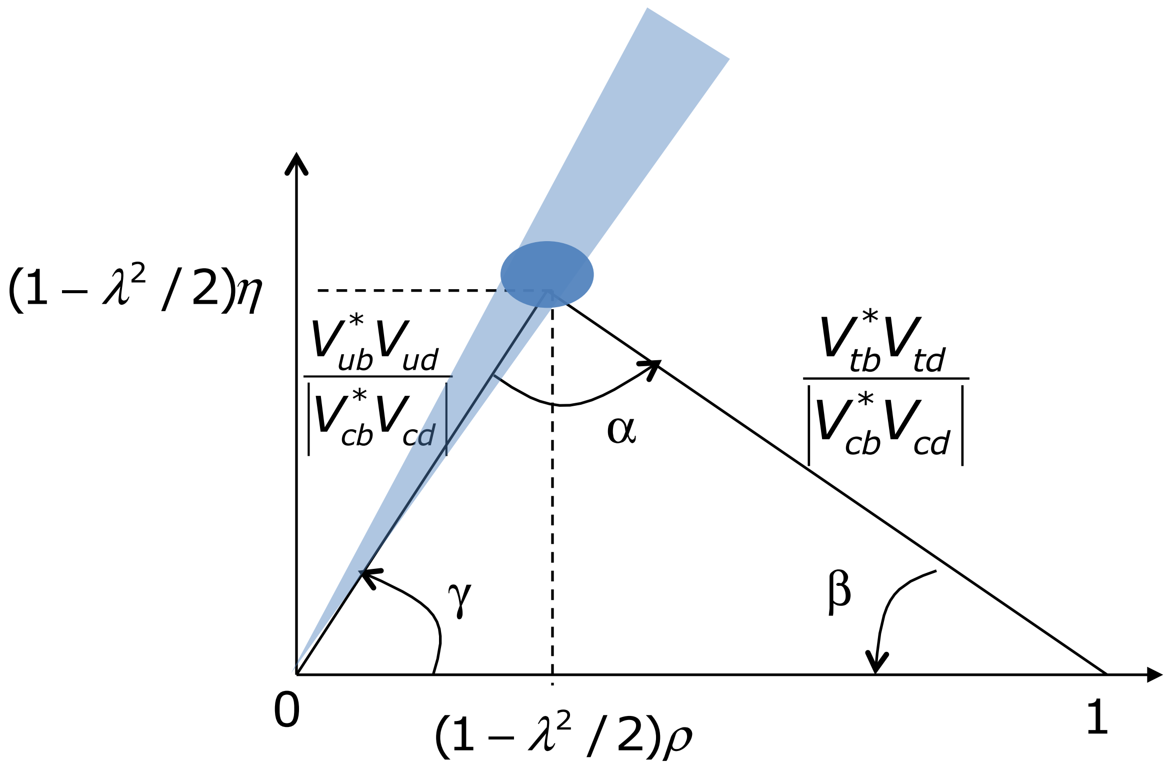
A lot of decay modes

⊗

Three different experiments (BaBar, BELLE & LHCb)

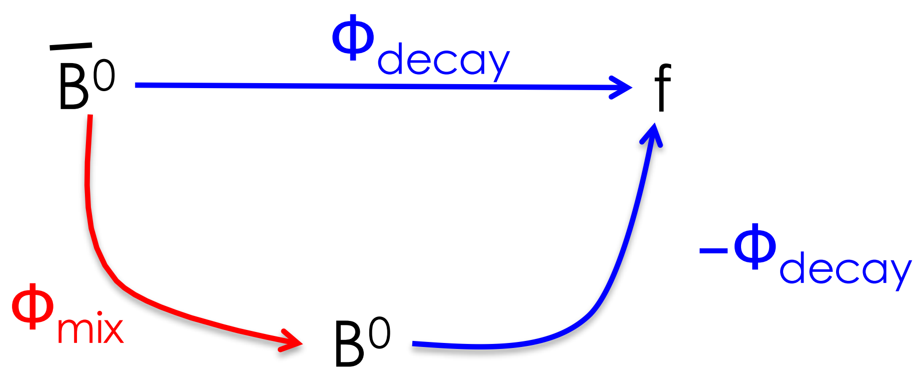



$\gamma = (70.5 \pm 5.7)^{\circ}$



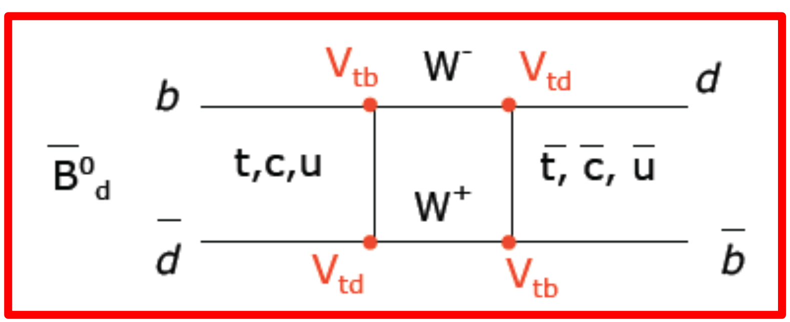
$(1-\lambda^2/2)\eta$
$\frac{V_{ub}^* V_{ud}}{|V_{cb}^* V_{cd}|}$
$\frac{V_{tb}^* V_{td}}{|V_{cb}^* V_{cd}|}$
α
γ
β
0
1
$(1-\lambda^2/2)\rho$

# An example of CP induced by the interference between and decay : the β angle

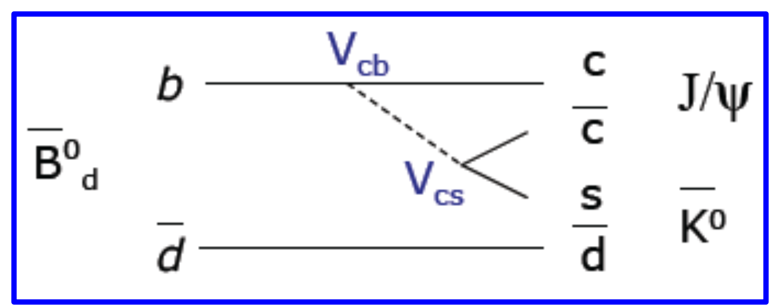



$$\Phi_d = \Phi_{mix} - 2\,\Phi_{decay}$$

Mixing



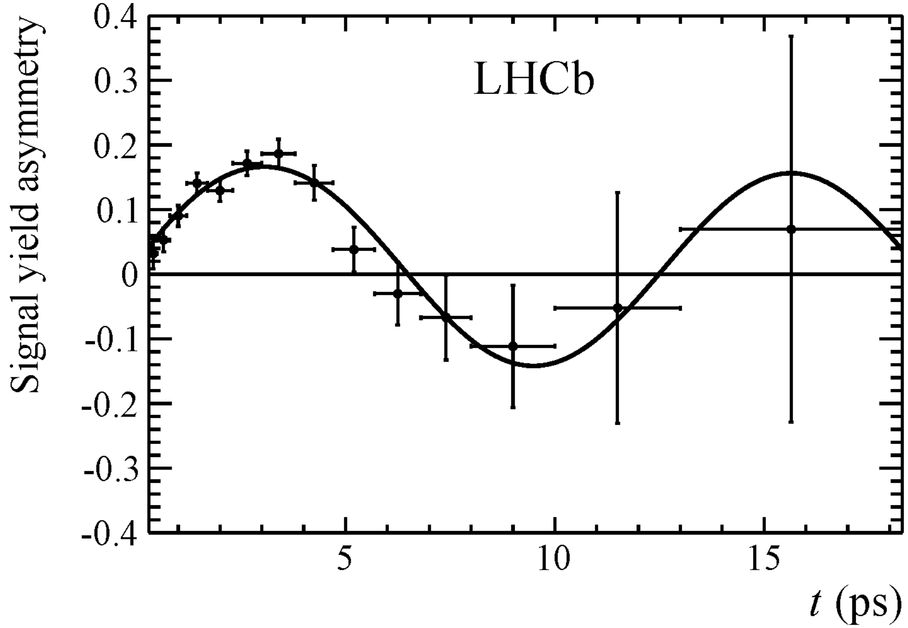

Decay



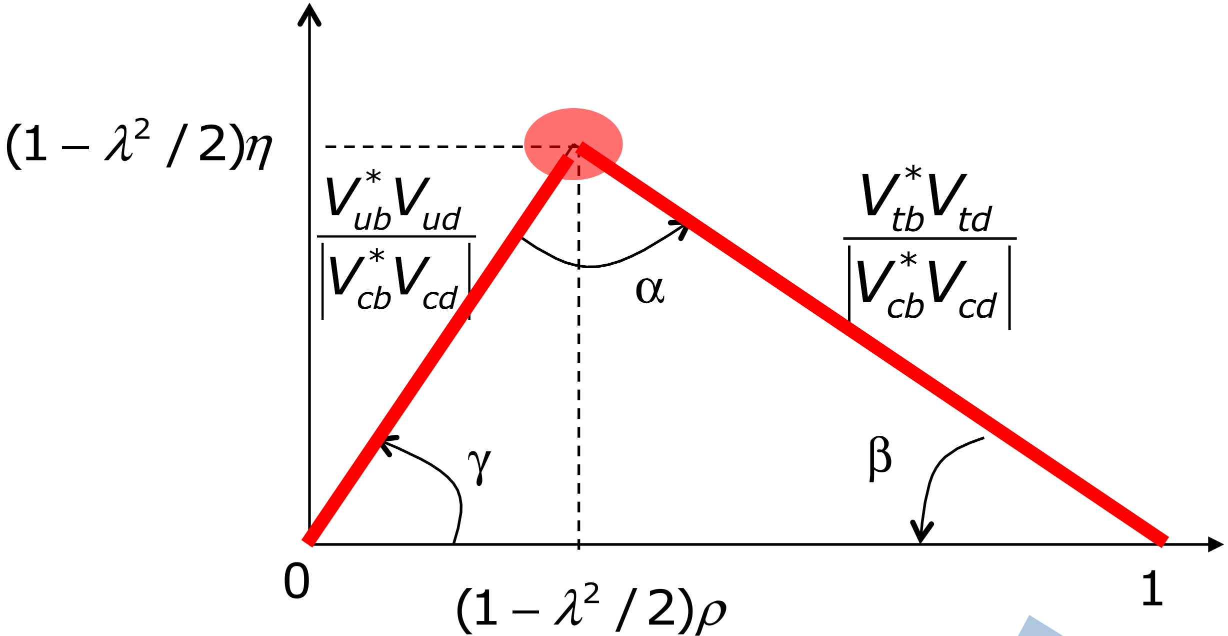
$$a_{f_{CP}}(t) = \frac{\text{Prob}(B^0(t) \to f_{CP}) - \text{Prob}(\overline{B^0}(t) \to f_{CP})}{\text{Prob}(\overline{B^0}(t) \to f_{CP}) + \text{Prob}(B^0(t) \to f_{CP})} =$$

$$= \sin(2\beta)\ \sin(\Delta m t)$$

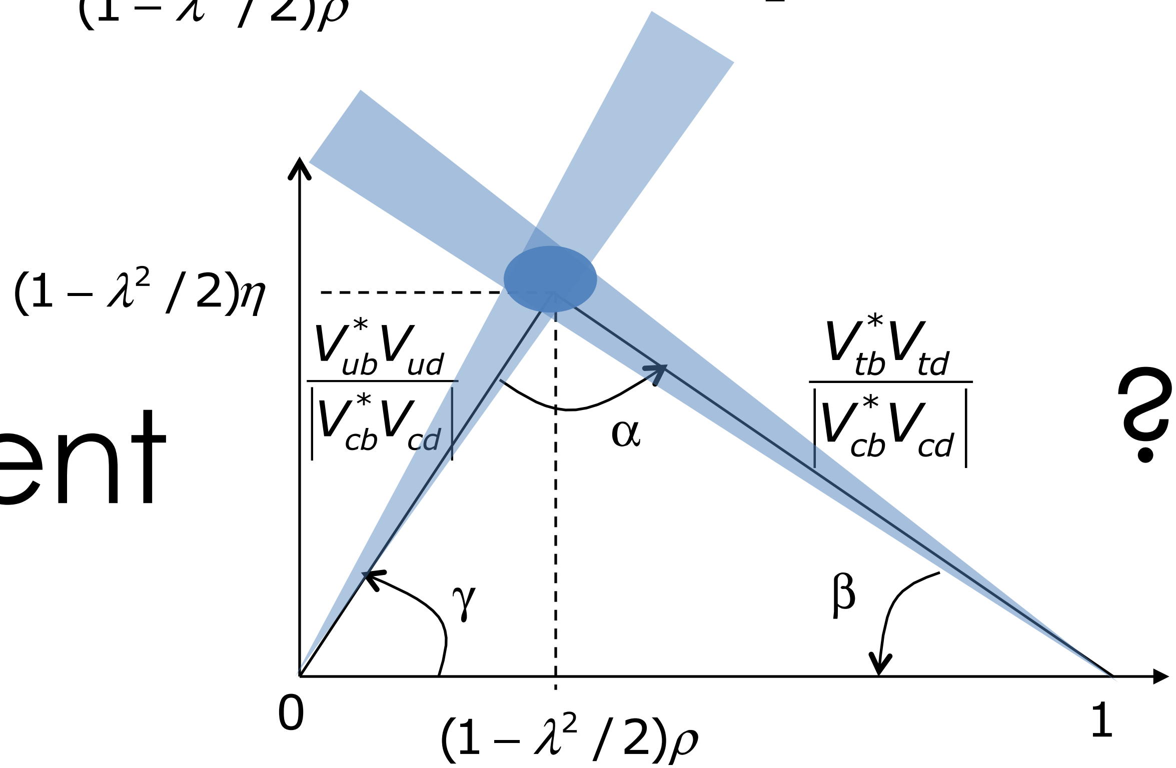
Pionnered by the B-factories



Is




in agreement with

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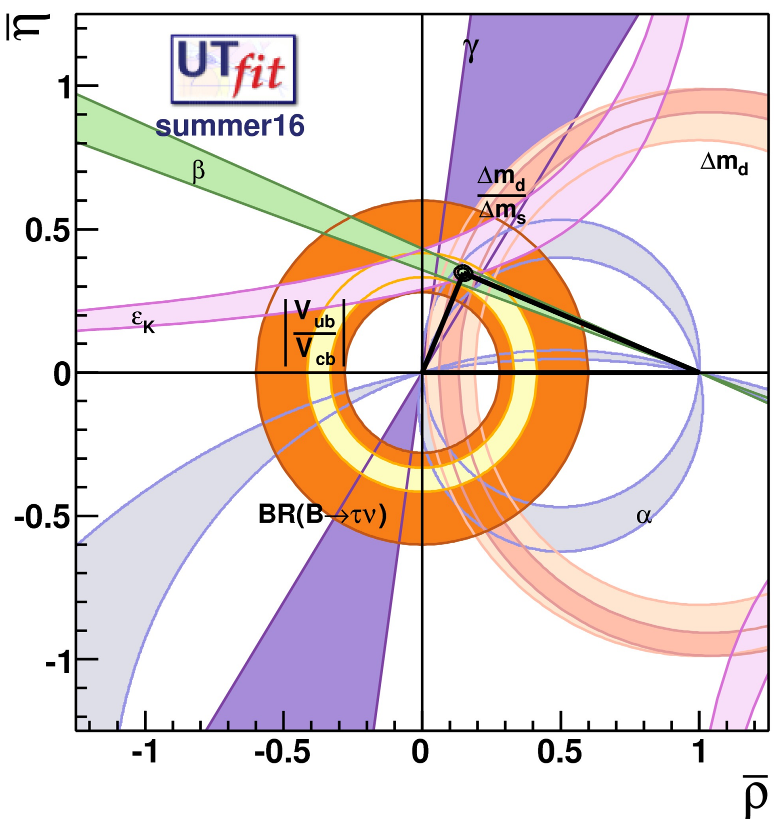
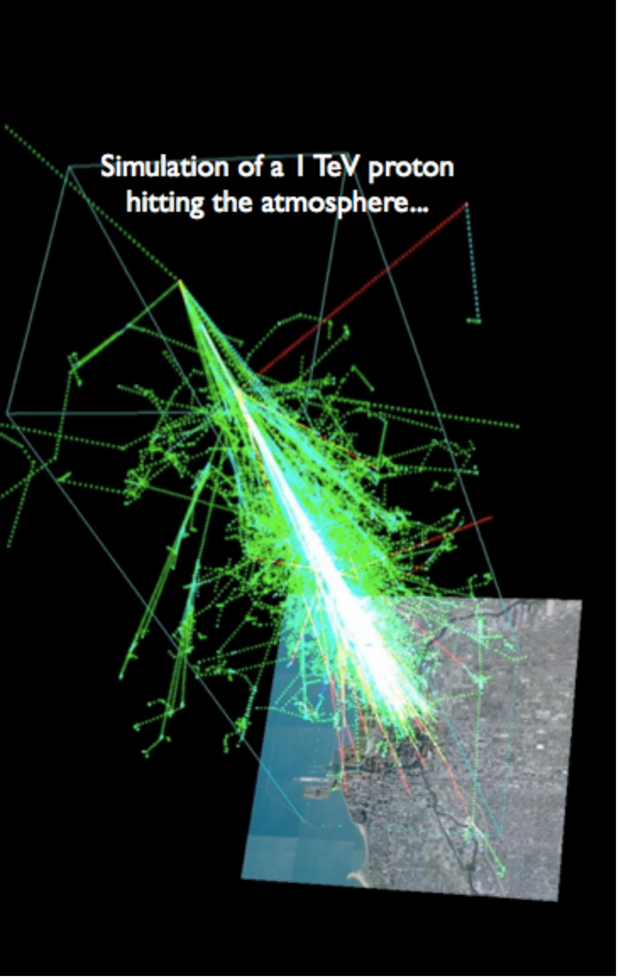



Sides and angles measurements in good agreement

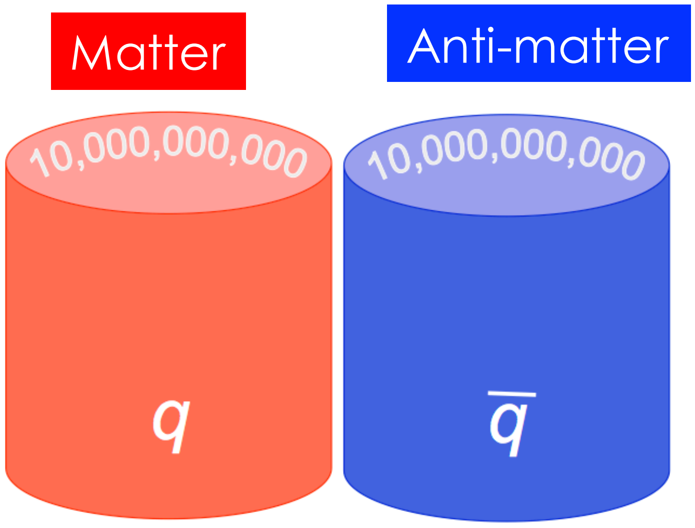
The CKM model of CP violation has been confirmed

At the electroweak scale, the CKM mechanism dominates CP Violation

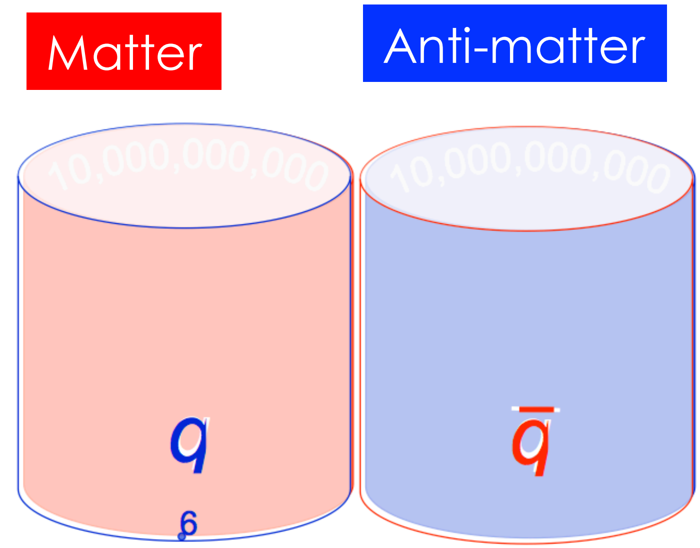



- Anti-matter in cosmic rays
- No sign of light emission (anti-galaxy …)
- No sign of anti-nuclei (anti-$He^4$ … )
Searches on-going

# Anti-matter in the Universe and Big Bang




Primordial Universe



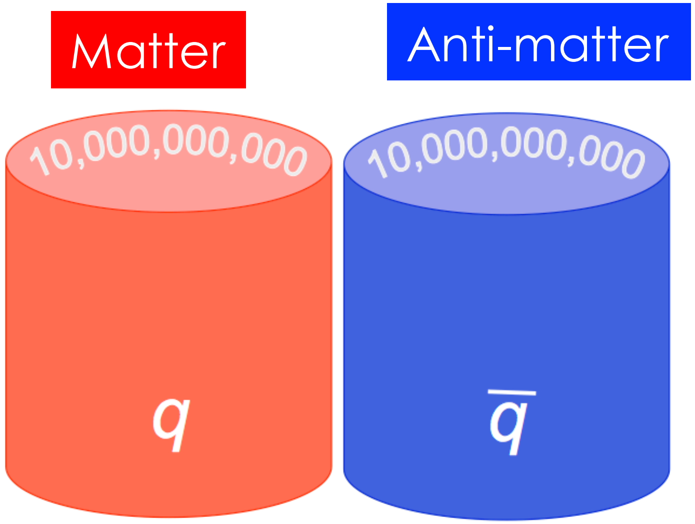

Today

$$\frac{n(baryon) - n(antibaryon)}{n_\gamma} \sim 6\ 10^{-10}$$

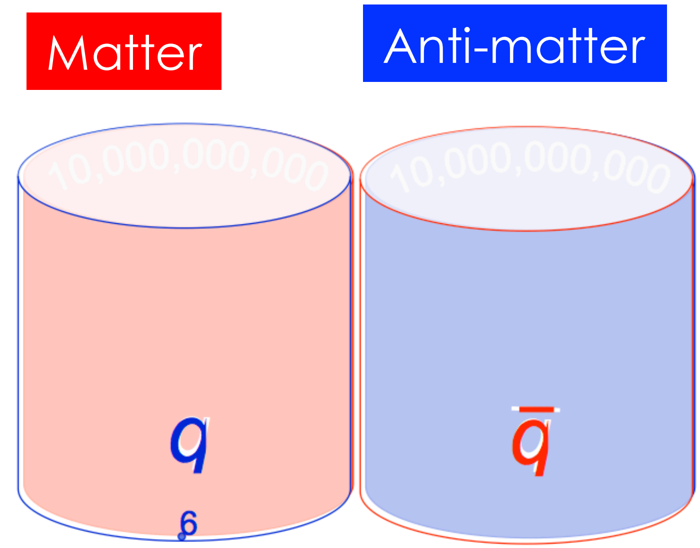
## The 3 Sakharov conditions(1967)

1. Baryonic number violation: $X \rightarrow p\, e^-$
2. C and CP symmetries violation: $\Gamma(X \rightarrow p\, e^-) \neq \Gamma(\overline{X} \rightarrow \overline{p}\, e^+)$
3. To be out of equilibrium: $\Gamma(X \rightarrow p\, e^-) \neq \Gamma(p\, e^- \rightarrow X)$

# Anti-matter in the Universe and Big Bang




Primordial Universe




Today

$$\frac{n(baryon) - n(antibaryon)}{n_\gamma} \sim 6\ 10^{-10}$$

## The 3 Sakharov conditions(1967)

1. Baryonic n

2. C and CP

3. To be out of equilibrium:

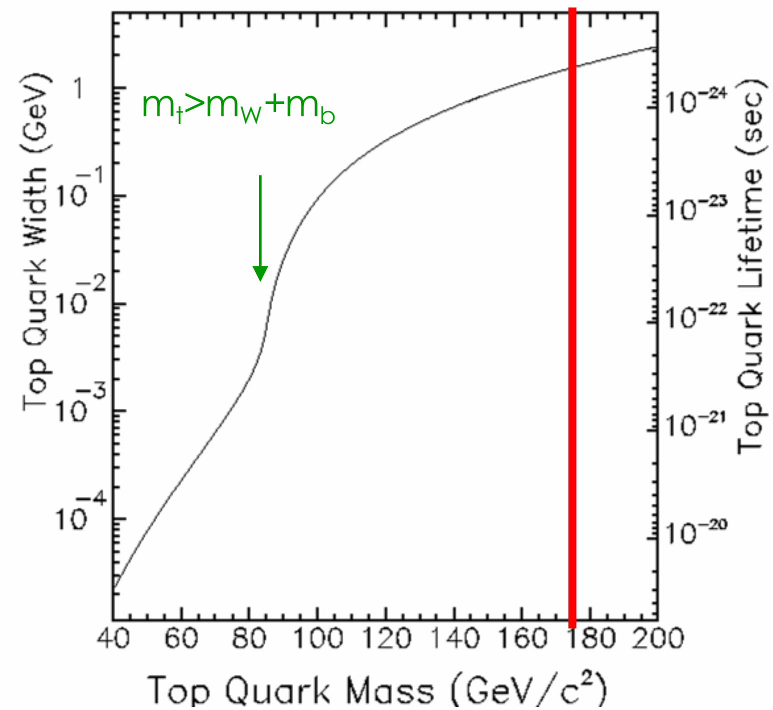
But the CP violation phase of the SM is orders of magnitude too small

(

Heavy Flavours

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Heavy flavours !



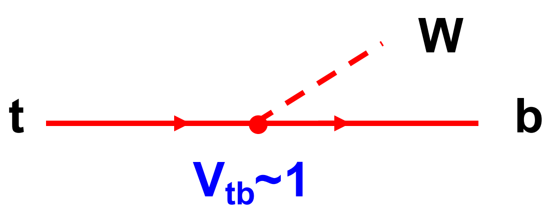

Phys. Lett. B 181 (157)

Why not the top quark ?

The decay $\propto m^5 \Rightarrow$ extremely short lifetime

Hadronization time ~$10^{-23}$ s

$\Rightarrow$ no top hadrons



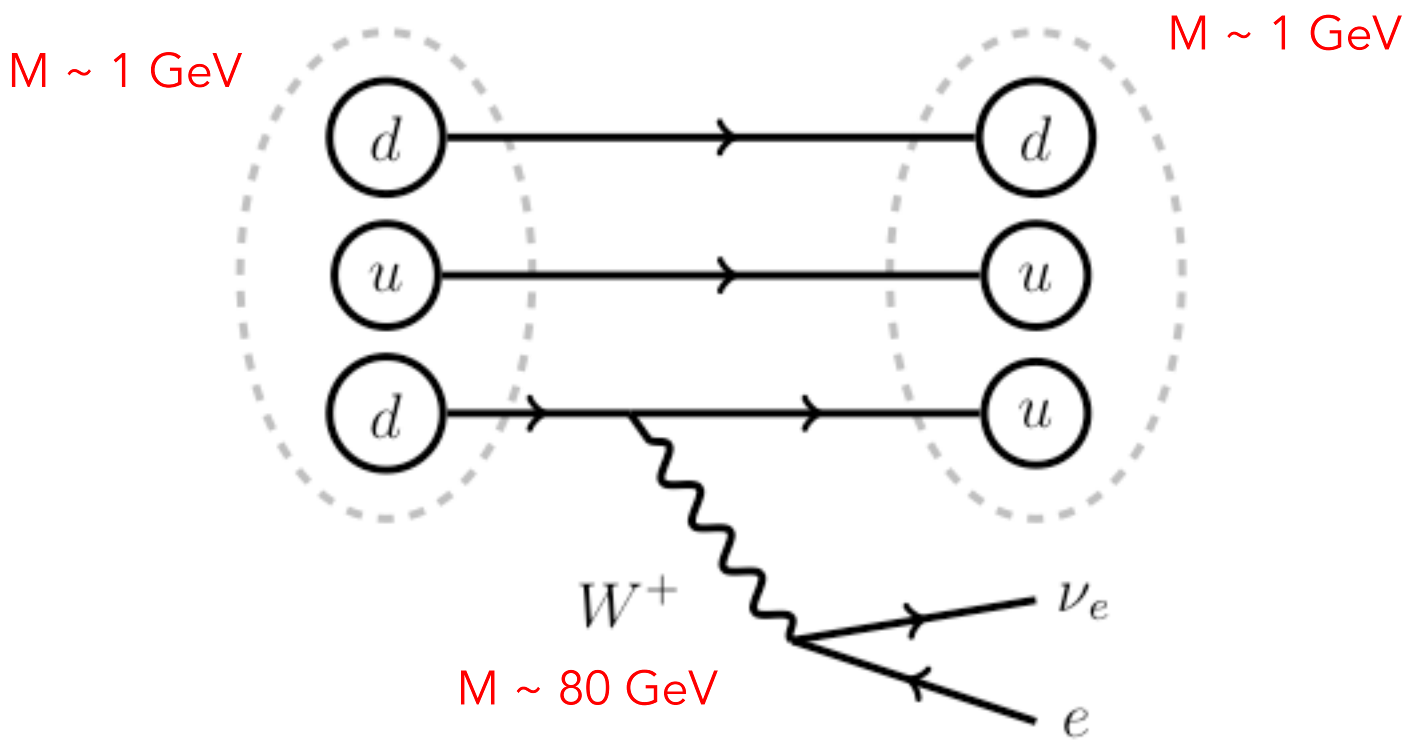

Tevatron + LHC :

$|V_{tb}| = 1.009 \pm 0.031$ [PDG]

# Heavy Flavours why ?

β decay of the neutron

Phenomena taking place at ~ 1 GeV reveals physics at the 100 GeV scale



# The top quark at an e+ e- collider with √s=10 GeV in 1987 !

$e^+ e^- \to \Upsilon(4S) \to \bar{B}B$ at $\sqrt{s}$ = 10.58 GeV

Argus Collaboration
Phys Lett B 192 p454

Production of coherent *BB* pairs

$B^0 \to D^{*-}\mu^+\nu$

$B^0 \to D^{*-}\mu^+\nu$

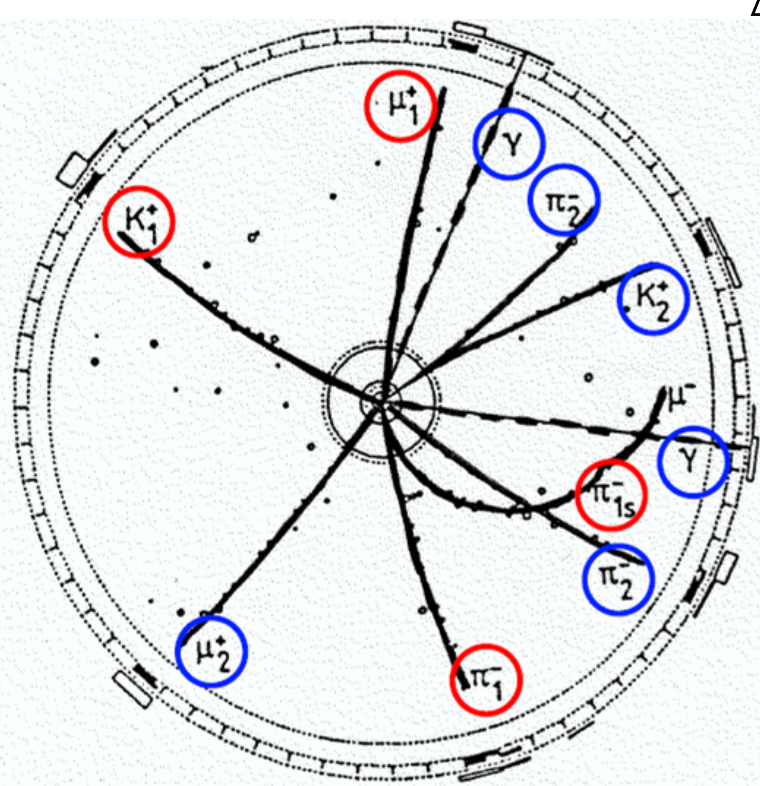



Fig. 11: The fully reconstructed ARGUS event [26] $e^+e^- \to \Upsilon(4S) \to B^0\overline{B}^0 \to B^0B^0$ as the first evidence for the occurance of $B^0\overline{B}^0$ oscillations.
$B^0 \to D_1^{*-}\mu_1^+\nu$,
$D_1^{*-} \to \pi_{1s}^-\overline{D}^0$, $\overline{D}^0 \to K_1^+\pi_1^-$.
$\overline{B}^0 \to B^0 \to D_2^{*-}\mu_2^+\nu$,
$D_2^{*-} \to \pi^0 D_2^-$,
$\pi^0 \to \gamma\gamma$, $D_2^- \to K_2^+\pi_2^-\pi_2^-$.

$$\Delta m_B \approx 0.00002 \cdot \left(\frac{m_t}{\mathrm{GeV}/c^2}\right)^2 \mathrm{ps}^{-1}$$

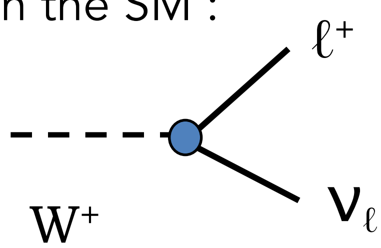
$$\approx 0.5\mathrm{ps}^{-1}$$

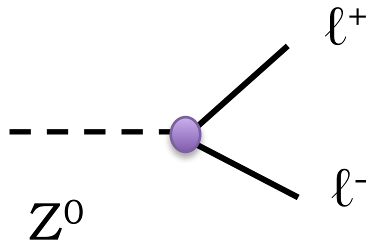
$\Rightarrow m_t > 50$ GeV

First hint of a *really large* $m_{top}$!

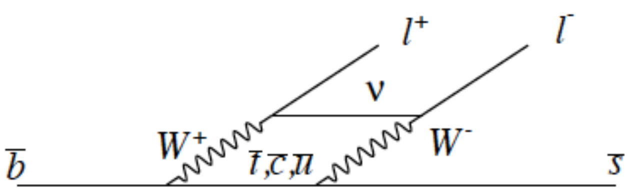
# Lepton Flavour Universality tests with B decays

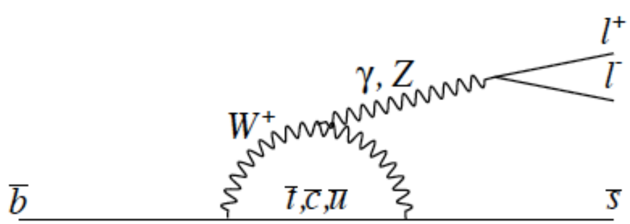
In the SM :




$\ell$=e, μ or τ



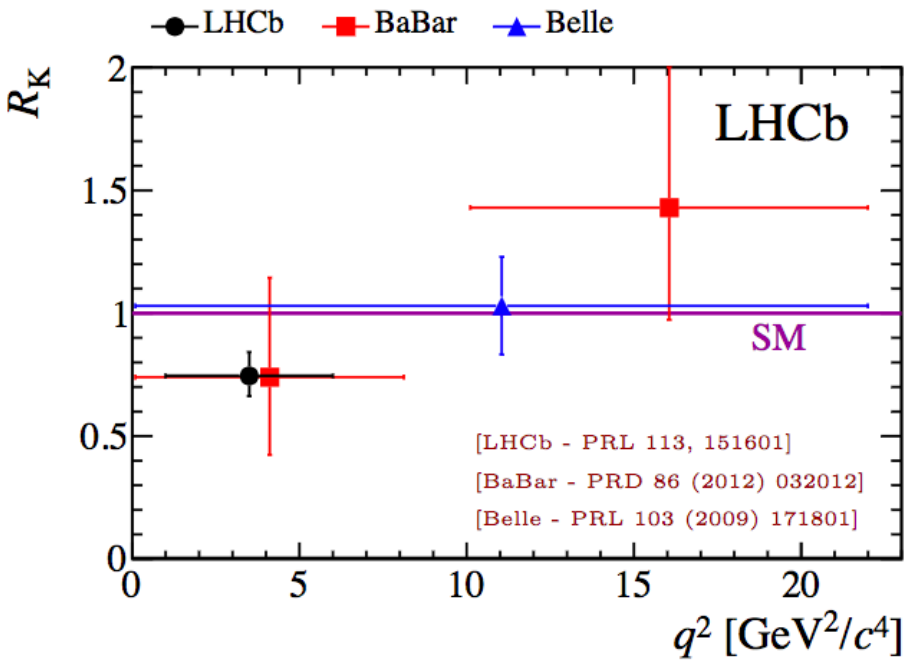

$$R(K) = \frac{B \to K\mu\mu}{B \to Kee}$$

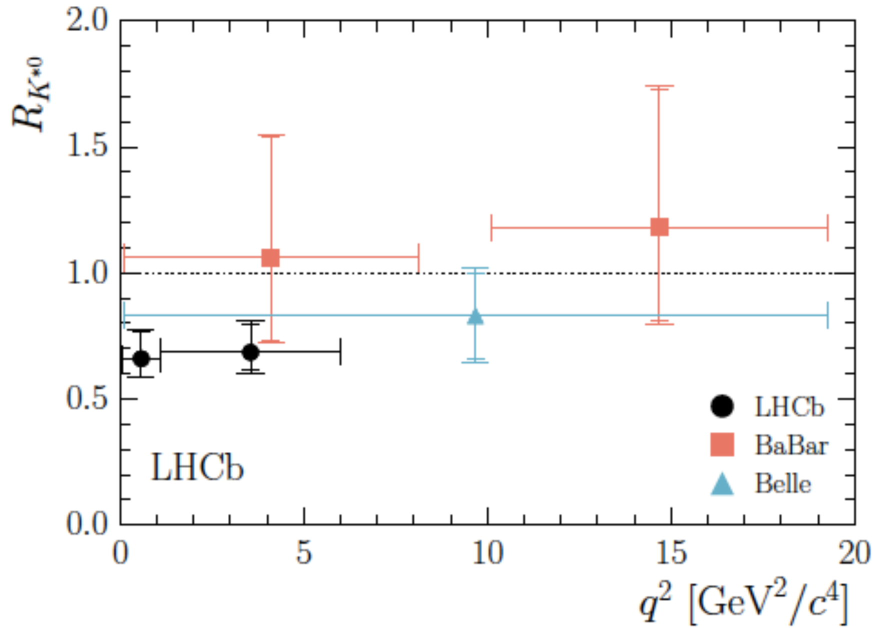
R=1 (at $10^{-3}$ ) in the SM

$$R(K) = \frac{B \to K\mu\mu}{B \to Kee}$$

$$R(K^*) = \frac{B \to K^*\mu\mu}{B \to K^*ee}$$

- JHEP08 (2017) 055
- PRD 86 (2012) 032012
- PRL 103 (2009) 171801




› Compatibility with the SM prediction(s)

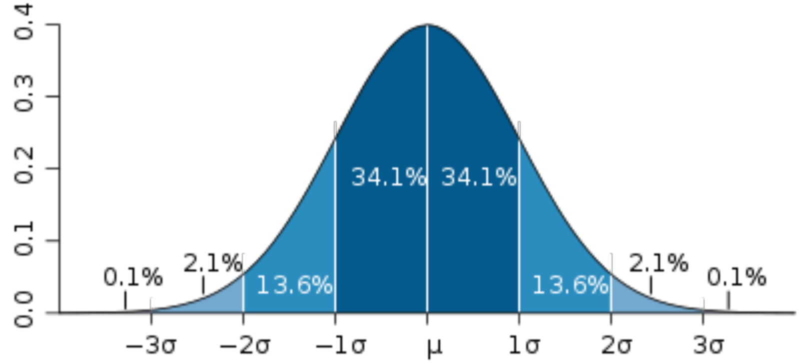
» $R_K$ 2.6σ

» $R_{K^{*0}}$ low-$q^2$ 2.1-2.3σ

» $R_{K^{*0}}$ central-$q^2$ 2.4-2.5σ

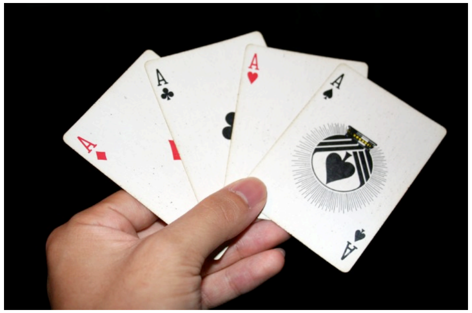
In particle physics there are some rules :
3 σ : evidence
5 σ : observation

# 3 or 5 σ ?




3 σ : probability, of 0.003
5 σ : probability, of $3\times10^{-7}$ (1 in 3.5 million)

*this is the probability to draw the 4 aces in a game of 52 cards in this order :*



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# Weak interaction in summary

- All quarks and leptons are sensitive to the weak interaction
- $M_W \sim M_Z \sim 100$ GeV → short range
- Extremely weak : (~ $10^{-8}$ smaller intensity than the strong interaction at a distance of 1 fm)

$$\sigma(\bar{\nu}p) \approx 10^{-43} cm^2 \quad E_\nu \sim 3\ MeV$$

- The weak interaction
  - violates maximally C and P
  - does not conserve the flavour
  - Exhibits a tiny CP violation
- The weak and mass eigenstates of quarks are not the same, they are related via $V_{CKM}$ which is a natural source of CP violation
- Heavy flavours is a privileged way to search for New Physics

and weak interaction means also neutrinos …
which we had very little time to discuss