The Weak Interaction

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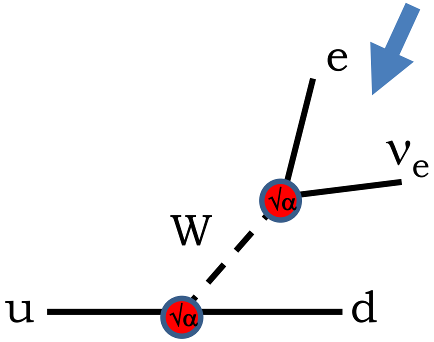
Introduction

1.1 Lifetimes (used an introductory example)

$\Delta^{++} \to p\pi$	~ 10^{-23} s	Strong interaction
$\Sigma^0 \to \Lambda\gamma$	~6 10^{-20} s	electromagnetic interaction
$\pi^0 \to \gamma\gamma$	~ 10^{-16} s	
$\Sigma \to n\pi$	~10^{-10} s	Weak interaction
$\pi^- \to \mu^- \bar{\nu}_\mu$	~10^{-8} s	
$\mu^- \to e^- \bar{\nu}_e \nu_\mu$	~10^{-6} s	
$n \to p\, e^- \bar{\nu}_e$	15 min	Beta decay



$$\Gamma \left(\propto \frac{1}{\tau} \right) \propto \text{coupling constant}$$

- How to explain these long lifetimes ?

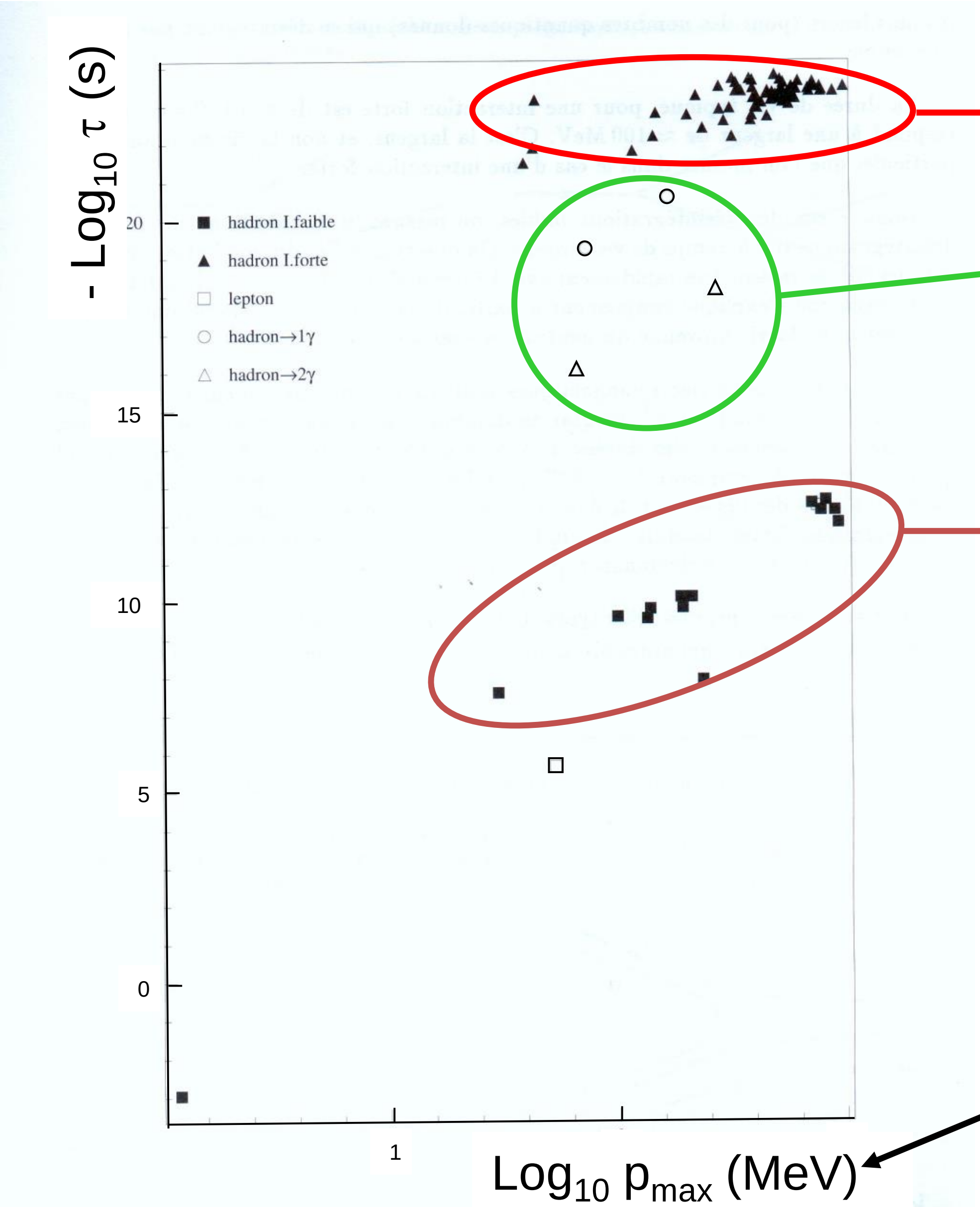
$$\frac{\tau(\Delta \to p\pi)}{\tau(\Sigma \to n\pi)} = \frac{10^{-23}}{10^{-10}} = 10^{-13} \approx \left(\frac{\alpha_W}{\alpha_s} \right)^2$$

$\Rightarrow$ Coupling for the weak interaction : $\alpha_W \sim 10^{-6}$

There is roughly the same phase space available ($M_n \sim M_p$ and $M_\Delta \sim M_\Sigma$)

In fact a full range of lifetimes has to be explain by the weak interaction (from 10^{-12} s to 15 min !).

Particles lifetime in function of the maximal momentum avaible for one of the decay product



hadrons strong interaction

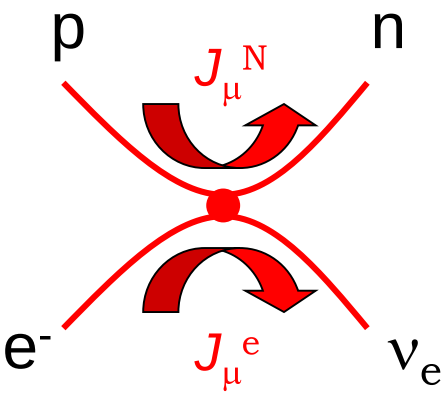
hadrons electromagnetic interaction

hadrons weak interaction

This quantity characterizes the available energy in the decay (phase space)

β decay and Fermi theory

1932 : Fermi proposes a theory which is the analogous of electromagnetism to explain the β decay for n→p $e^-\nu_e$ and p→n $e^+\nu_e$ a local interaction :



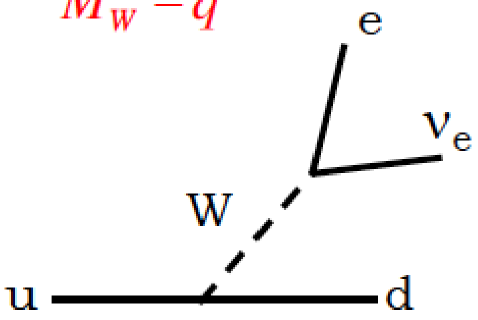
$$G\left(\bar{u}_n\gamma^\mu u_p\right)\left(\bar{u}_{\nu_e}\gamma_\mu u_e\right)$$

[GeV-2]

Through the analogy with electromagnetism, the G constant should be of GeV-2 dimension

We know today that the propagator is $\frac{1}{M_W^2 - q^2}$

So Fermi theory is valid for $q^2 << M_W^2$



$[G] = \mathrm{GeV}^{-2}$ $[G^2 m^5]=[\mathrm{GeV}]=[\mathrm{s}^{-1}]$

From dimensional arguments $\Gamma \sim G^2 E^5$

From precise muon lifetime

From calculations :

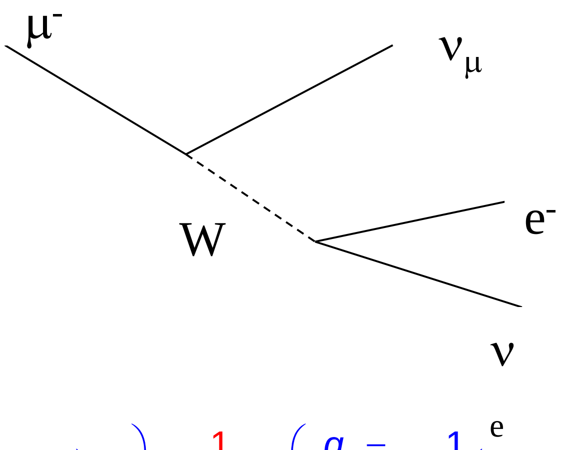
$$G_F = 1.16 \times 10^{-5}\, GeV^{-2}$$

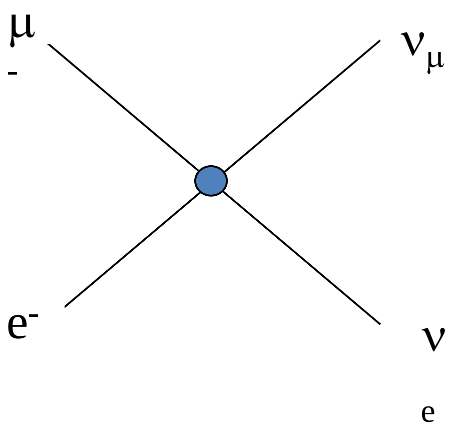
$$\mu^- \rightarrow e^- \ \bar{\nu}_e \ \nu_\mu \qquad \Gamma_\mu = \frac{G_\mu^2 m_\mu^5}{192\pi^3}$$

$G \sim 10^{-5}/M^2_N$

Weak EFFECTIVE coupling G_F : ~ 10^{-5}

Little deeper… try to undertand where the weakness of the weak interaction comes from…





SM gives that

$$M = \left(\frac{g}{\sqrt{2}} \bar{u}_{\nu_\mu} \gamma^\mu \frac{1}{2} (1-\gamma_5) u_\mu \right) \frac{1}{M_W^2 - q^2} \left(\frac{g}{\sqrt{2}} \bar{u}_e \gamma_\mu \frac{1}{2} (1-\gamma_5) u_{\nu_e} \right)$$

if $q^2 << M_W^2$ (which is the case for β decay for example)

$$M \sim \frac{g^2}{8M_W^2} \left(\bar{u}_{\nu_\mu} \gamma^\mu (1-\gamma_5) u_\mu \right) \left(\bar{u}_e \gamma_\mu (1-\gamma_5) u_{\nu_e} \right)$$

$$M \sim \frac{G_F}{\sqrt{2}} \left(\bar{u}_{\nu_\mu} \gamma^\mu (1-\gamma_5) u_\mu \right) \left(\bar{u}_e \gamma_\mu (1-\gamma_5) u_{\nu_e} \right)$$

$$\boxed{\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}}$$

But what is the value for M_W?

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2} \qquad \Rightarrow M_W = \left(\frac{g^2\sqrt{2}}{8G_F}\right)^{1/2}$$

Under the hypothesis $g \sim e$ and $G_F = \frac{10^{-5}}{M_p^2}$ with $e^2 = \frac{4\pi}{137}$

$M_W \sim 37$ GeV … large !

In fact $e = g\sin(\theta_W)$ and thus $M_W \sim 37\ \text{GeV}/\sin(\theta_W) \sim 80$ GeV

The weak interaction is not weak because of *g<<e* but because of the large value for the W mass (which is very different of what happens with QED and the photon)

$$G_F = \left(\frac{\sqrt{2}\ 4\pi\alpha}{8M_W^2\sin^2\theta_W}\right) \sim \frac{0.07}{M_W^2} \sim 1.2\times10^{-5}$$

Short distance force : R= $c\Delta t = \frac{\hbar}{M_W c} \approx 10^{-3}$ fm

Quantum numbers : conservation, non-conservation

- All particles which do not decay via strong or electromagnetic interactions will decay via weak interaction

  Briefly :
- Some rules work for all interactions :
  - Baryon number conservation
  - Lepton number conservation
  - Electric charge conservation

- Weak interaction violates :
  - The parity P
  - The charge conjugation C
  - CP
  - The isospin I
  - The strangeness S

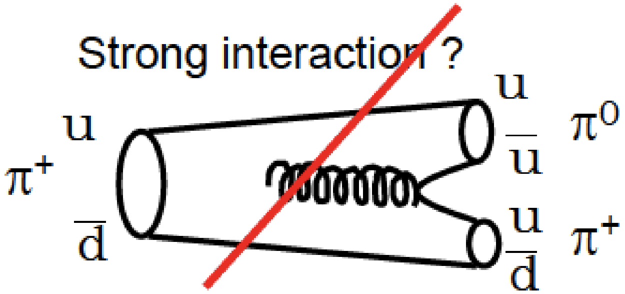
Discussed by M.H. Schune in the strong interactions

Allowed or forbidden ?

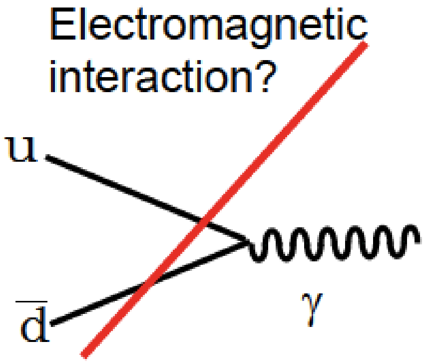
Example of the π+ decay (lighest hadron)

Experimental observation :
$\pi^0 \rightarrow \gamma\gamma$ (electromagnetic interaction)
$\pi^+ \rightarrow \mu\nu$ (weak interaction)

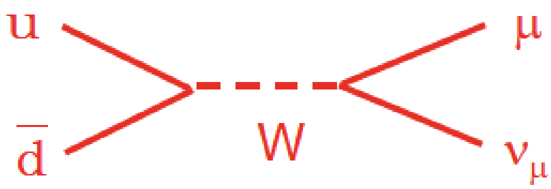
Why ???



Allowed for the ρ (M~770MeV). The π mass is too small



This coupling does not exist (electric charge conservation)



The only possibility is the weak interaction !

The θ-τ puzzle

Observation : two decays via the weak interaction

$$\tau \to \pi^+ \pi^+ \pi^-$$

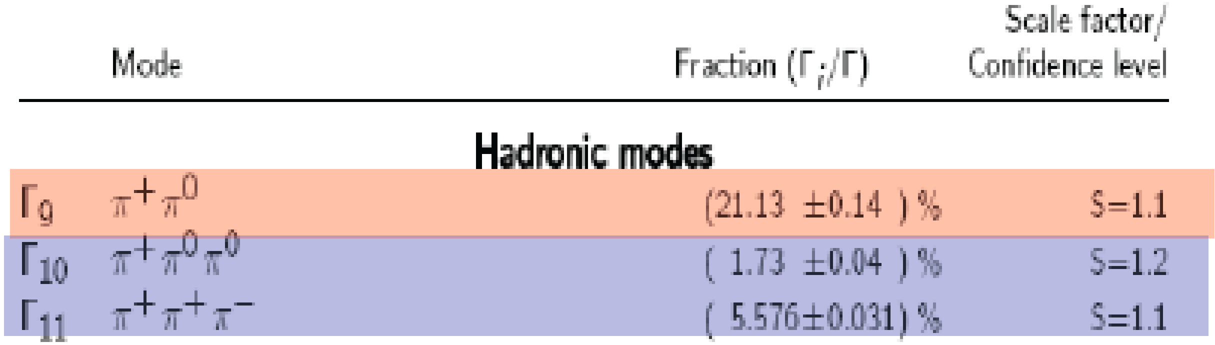
(this is not the tau lepton !)

$$\theta \to \pi^+ \pi^0$$

Experimentally : same mass, same lifetime

2 π : Parity = +1

3 π : Parity = -1



	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Hadronic modes			
Γ_9	$\pi^+\pi^0$	(21.13 ±0.14) %	S=1.1
Γ_{10}	$\pi^+\pi^0\pi^0$	(1.73 ±0.04) %	S=1.2
Γ_{11}	$\pi^+\pi^+\pi^-$	(5.576±0.031) %	S=1.1

Different values of the parity …

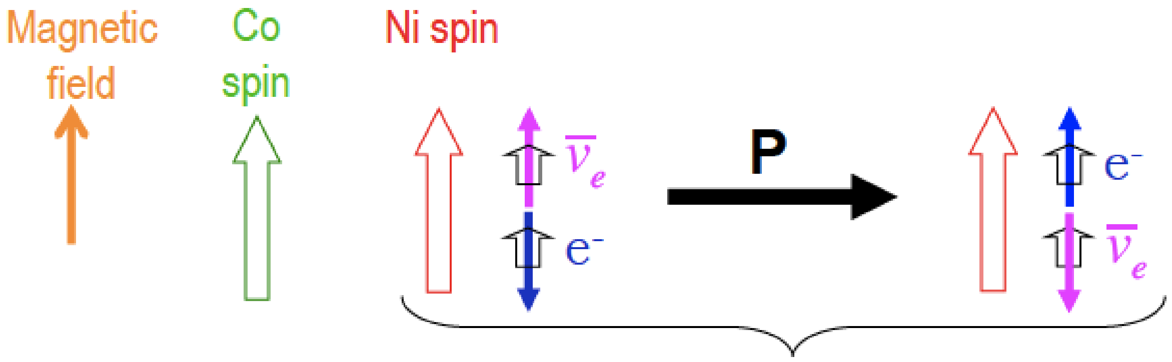
Under the hypothesis of parity conservation in the decay they cannot be the same particles !

Hypothesis to be tested experimentally !

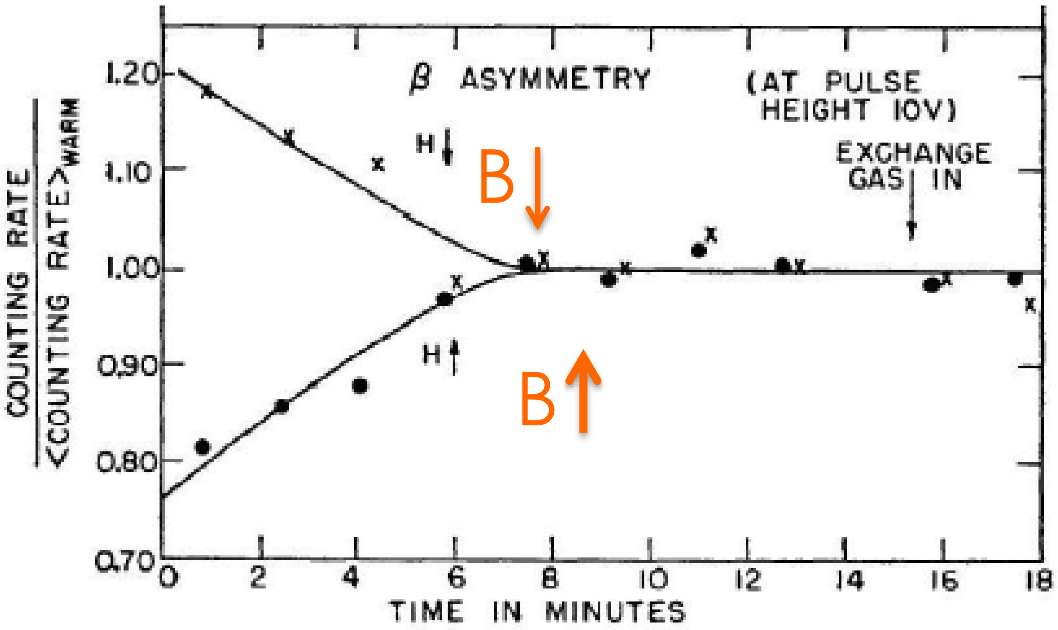
… → The Wu experiment

Schematical overview of the Co^{60} experiment

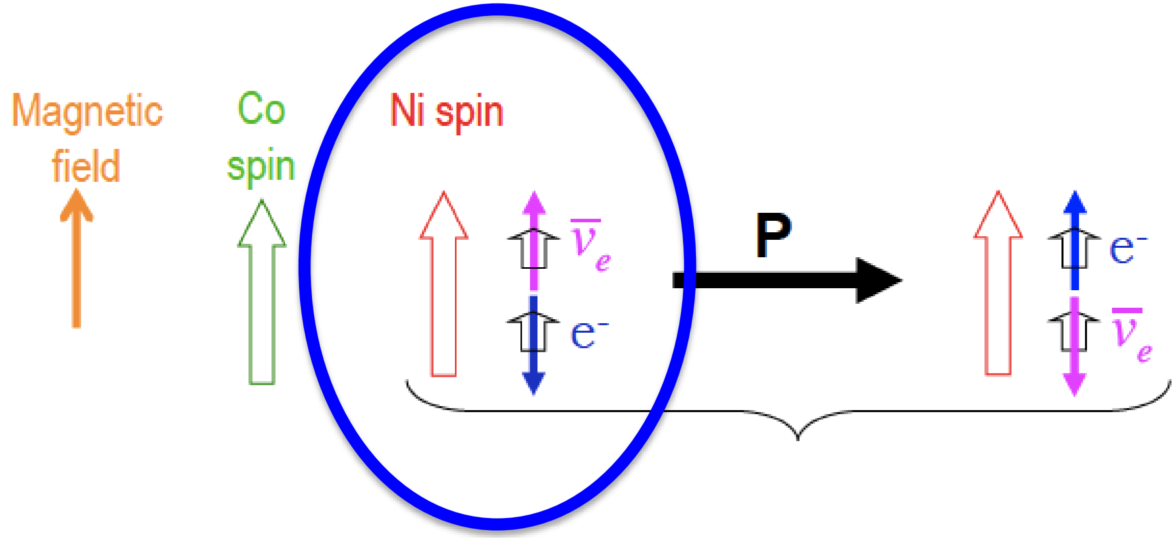
- β decay : $\mathrm{Co}^{60}\ (J=5) \rightarrow \mathrm{Ni}^{60*}\ (J=4)\, e^{-} \bar{\nu}_e$ $\quad \mathrm{n} \rightarrow \mathrm{p}\, e^{-} \bar{\nu}_e$
- Wu's experiment :
  - The spin of the Co^{60} atoms are aligned by a magnetic field
  - Record of the direction of the emitted electrons



If P is conserved these two configurations should have the same probability

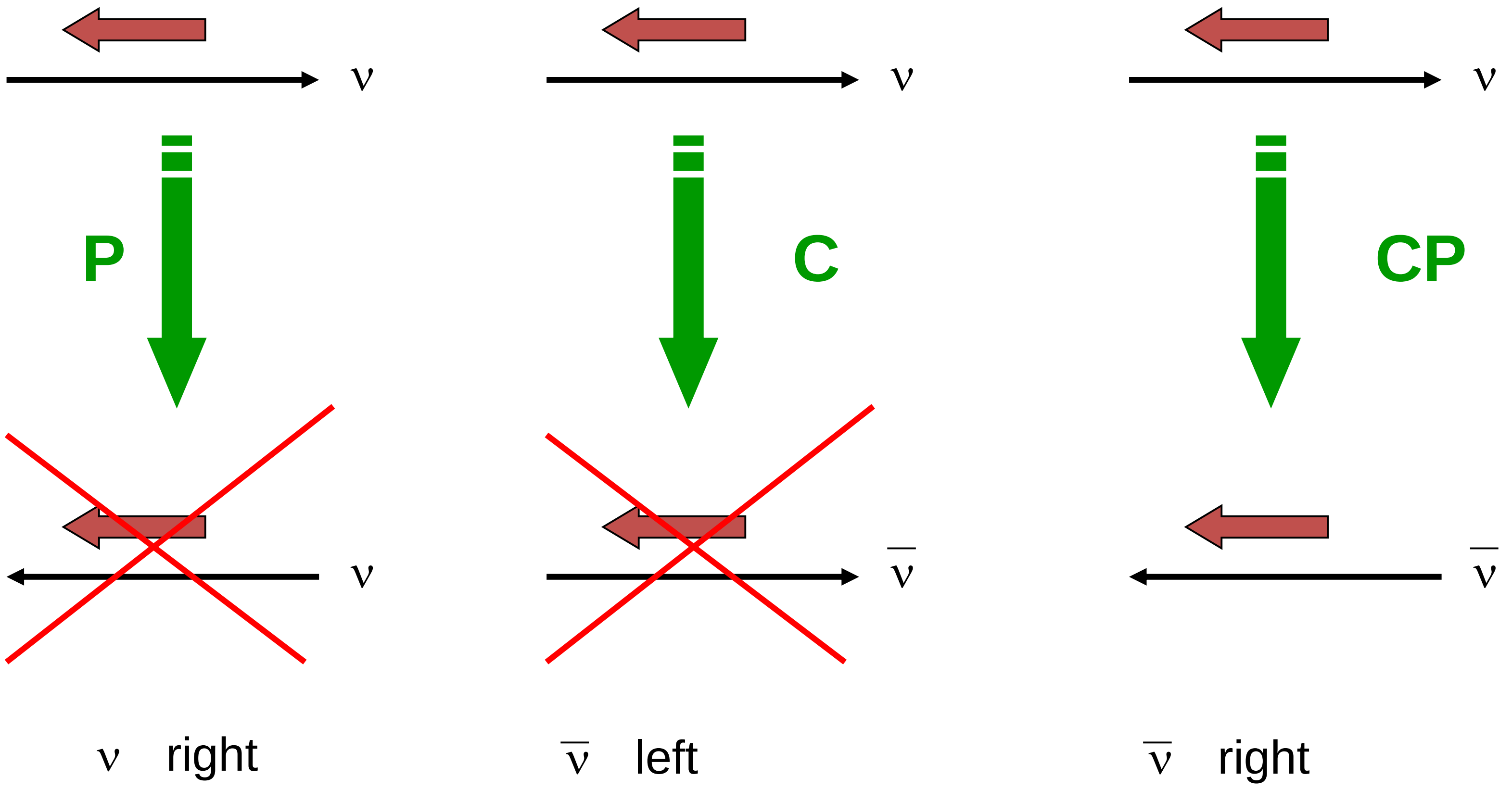


Result of the experiment: the e^- are preferentially emitted in the direction **opposite** to the Co spin (asymmetry)



In addition with other experiment (Goldhaber *et al* experiment) :

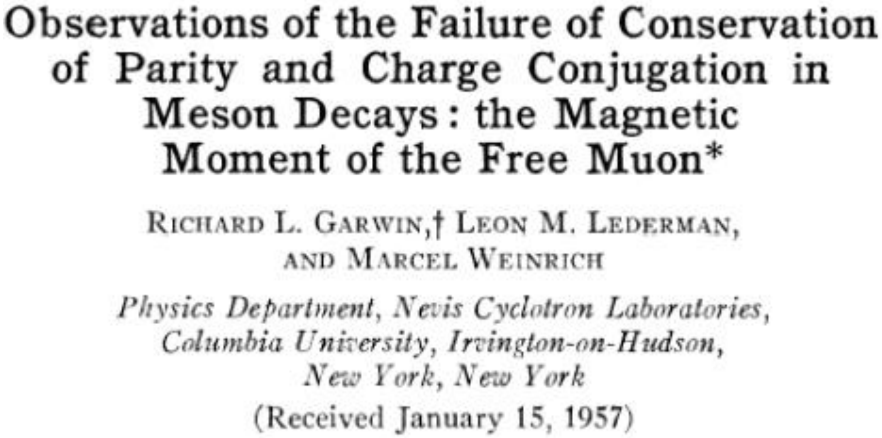
C et **P** are automatically violated in weak decays involving neutrinos :



One sees that the anti-particles helicity is the opposite of the particles helicity.

The ν is left handed (the anti-neutrino is right handed)

P, C and CP



Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

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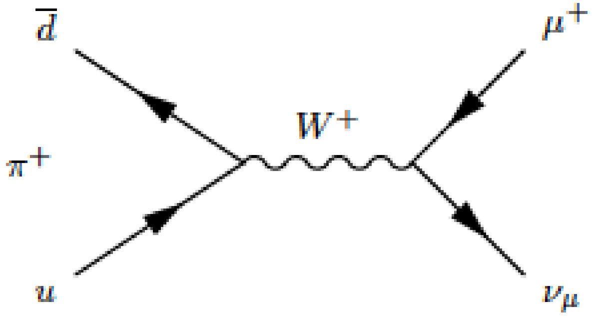
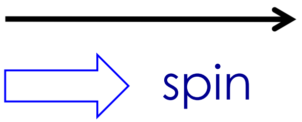
(Received January 15, 1957)

$\pi^+ \to \mu^+ \nu_\mu$

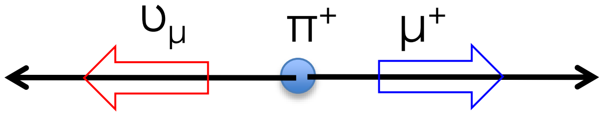


Spin of the pion : 0
Spin of the muon and neutrino : ½

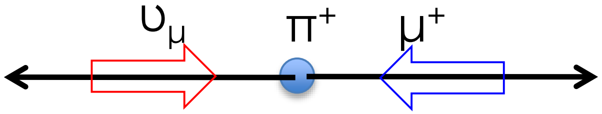


Momentum

spin



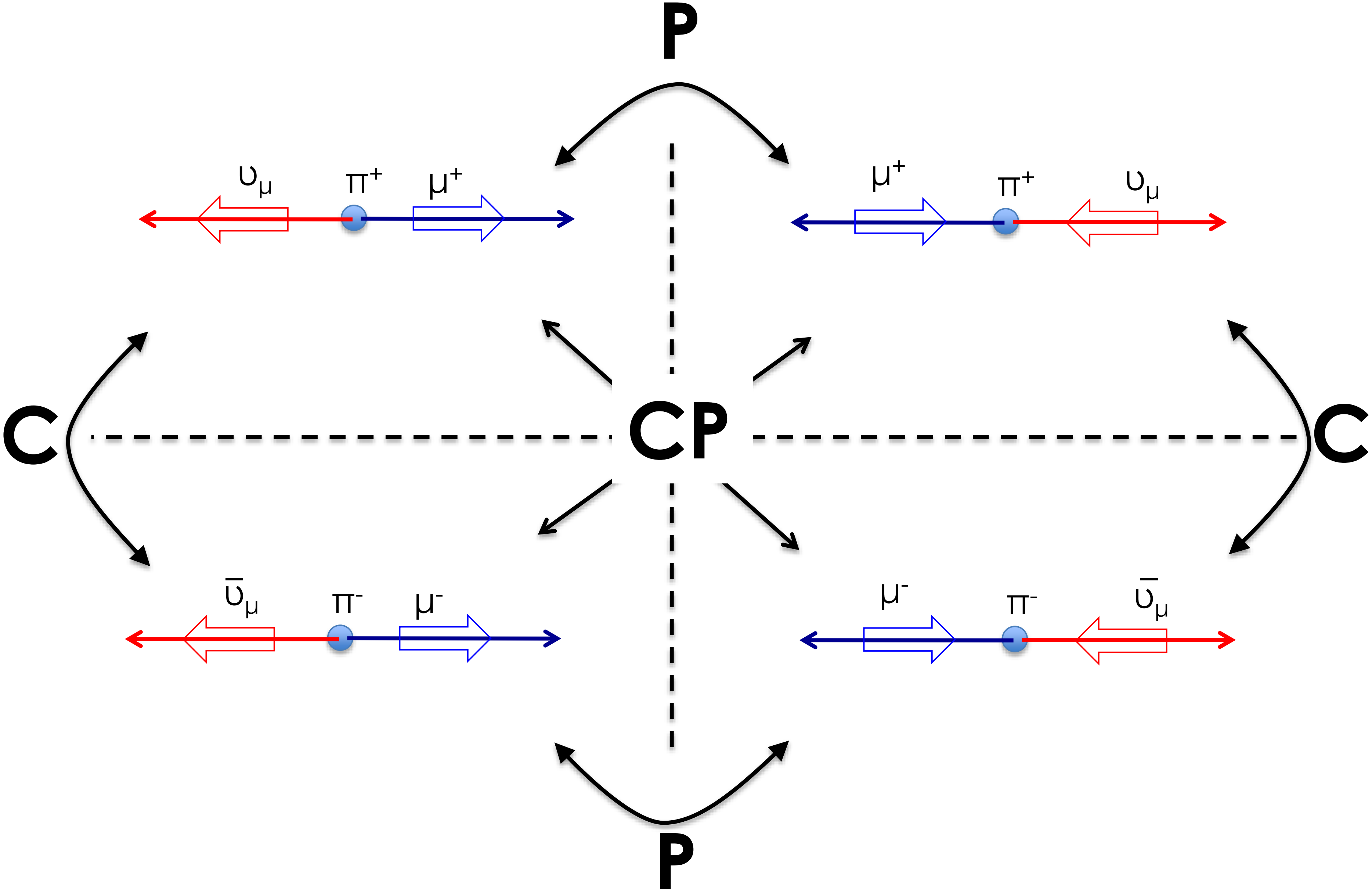
Right handed neutrino



Left handed neutrino

→ Spin of the decay products : oppositely aligned

→ Helicity of the neutrino is the same as that of the muon



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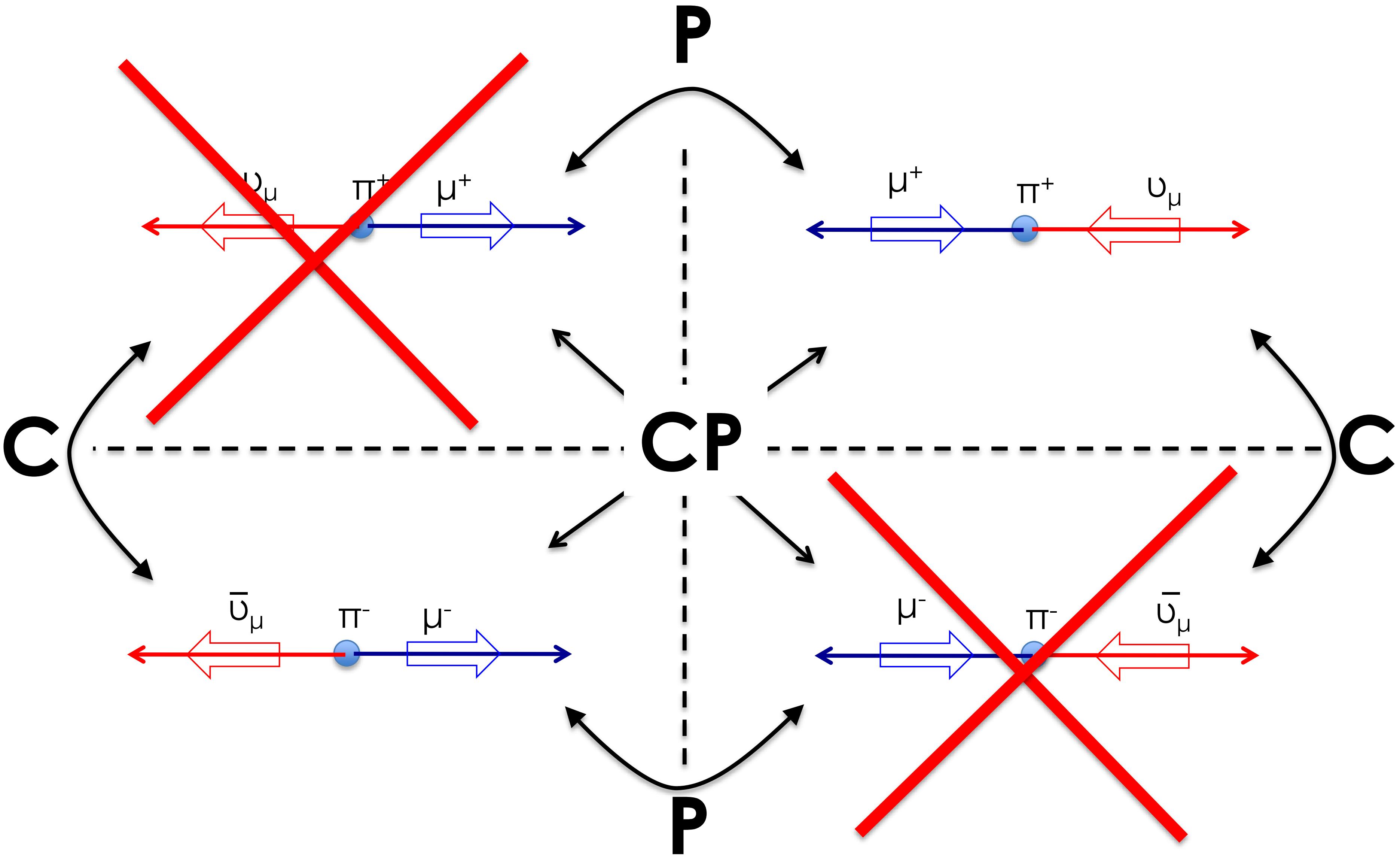


The ν is left handed (the anti-neutrino is right handed)

Measurement of the neutrino helicity (Goldhaber exp.)

From the stange particles discovery to charm, neutral current and the Cabibbo Matrix

~ 1947 : ‘strange’ particles discovered in cosmic rays : K(500 MeV), Λ(1100 MeV)

- the K and Λ production cross sections are similar to those of the other known hadrons of that time (the pion …)
- Their lifetime : of the order of 10^{-10} s (much longer than the time scale of the strong interaction (10^{-23} s)
- => different interaction in the decay !

Proposed explanation :

- They are produced by the strong interaction but decay via another interaction (the weak interaction)
- But why don’t they decay via the strong of leectromagnetic interaction ?
- Something should forbid it !

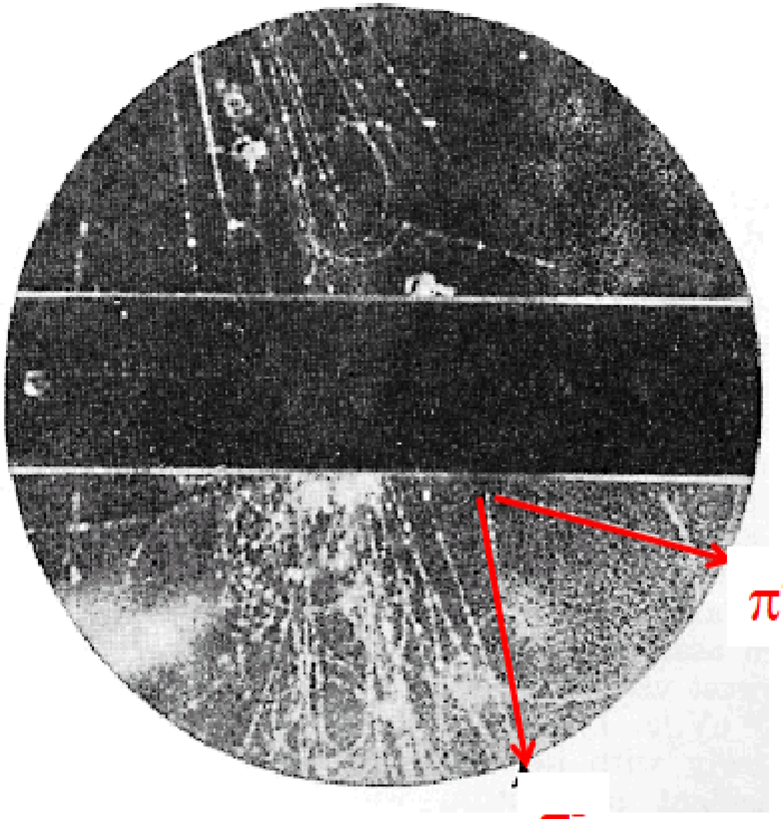
Pais’s intuition (1952) :

A new quantum number conserved by the strong interaction and violated by the weak interaction : strangeness

Experimental signs of strange particles : the K

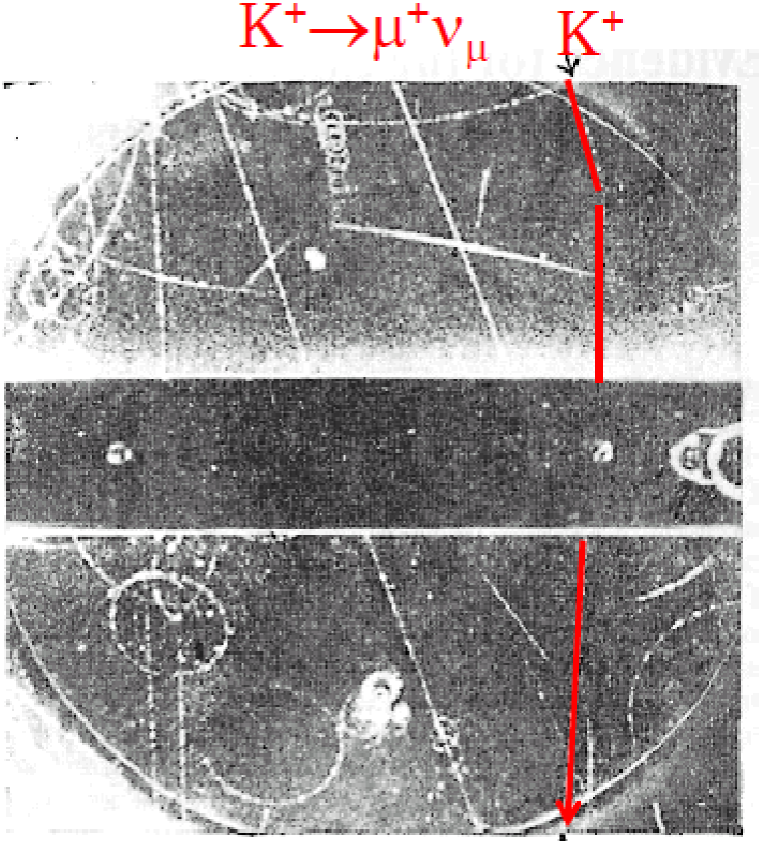
Cloud chamber ~1947

$K^0 \rightarrow \pi^+\pi^-$



V-particle

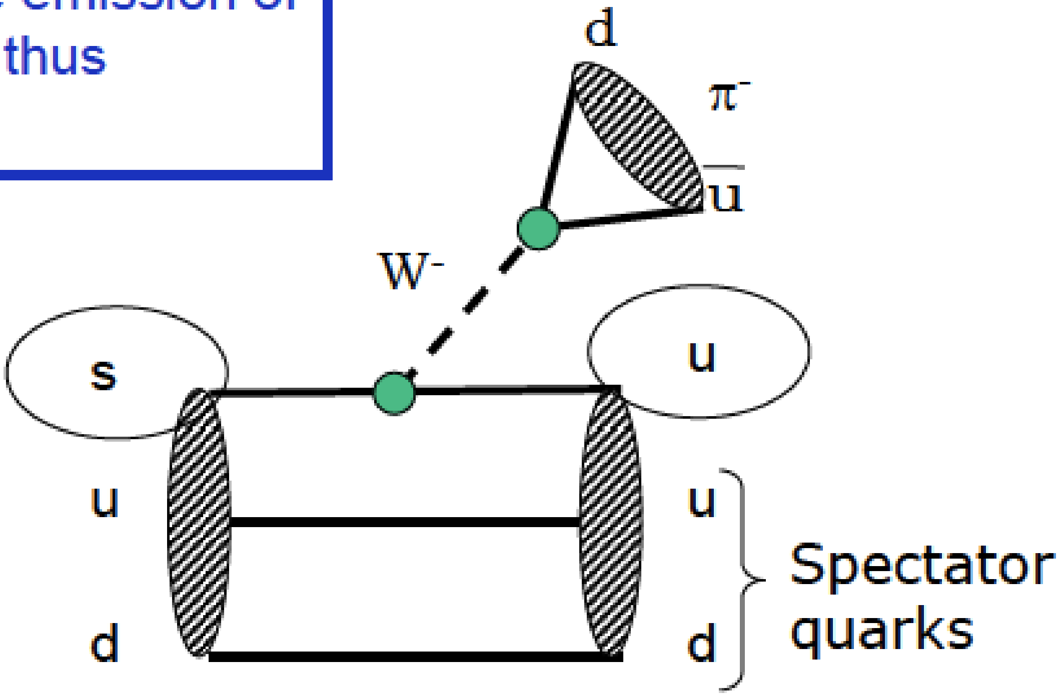
$K^+ \rightarrow \mu^+\nu_\mu$



«Kink» in the detector

Non conservation of strangeness :

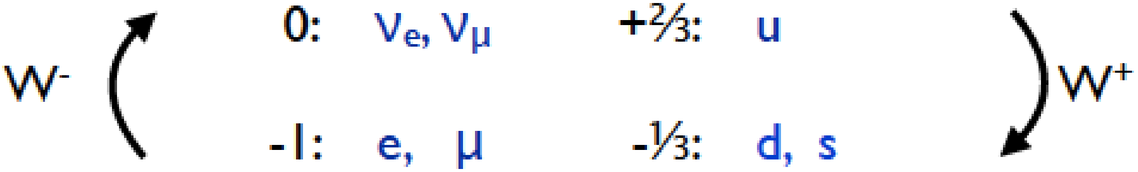
The s quark changes into a u quark through the emission of a W : S and I are thus violated.



In the 60's :

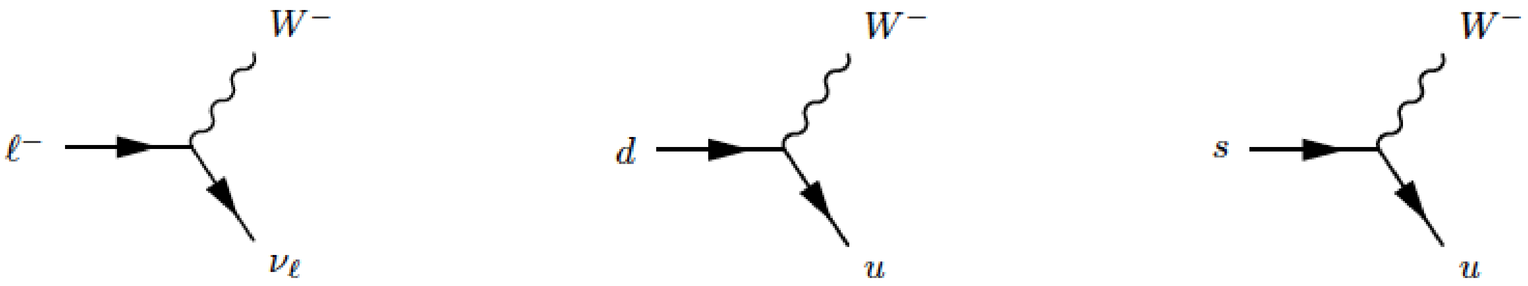
4 types of leptons : $e, \upsilon_e, \mu, \upsilon_\mu$

3 types of quarks : u, d, s (but not fully accepted)



But using the coupling extracted from the muon lifetime to predict the neutron lifetime or the lifetime of the strange particles (containing an s-qaurk) does not work well…

Muon lifetime

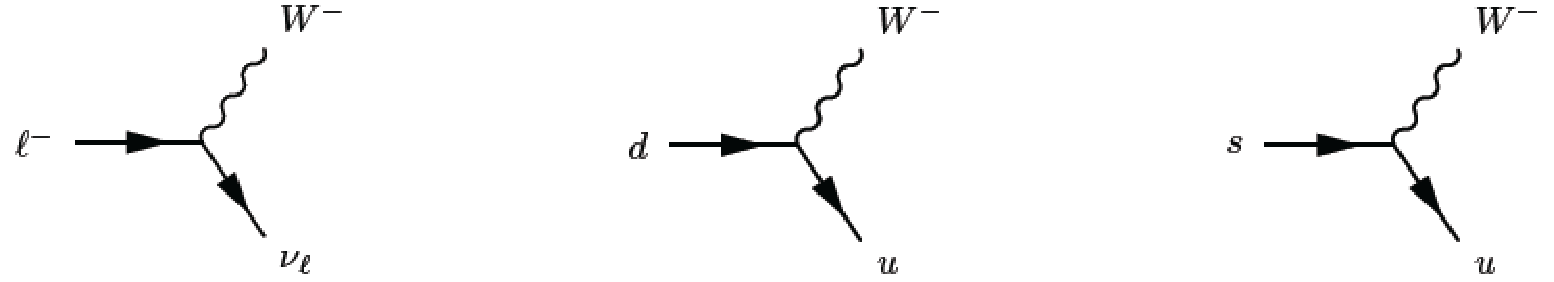


$$g \quad > \quad g' \quad >> \quad g''$$

Different couplings ????

UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)



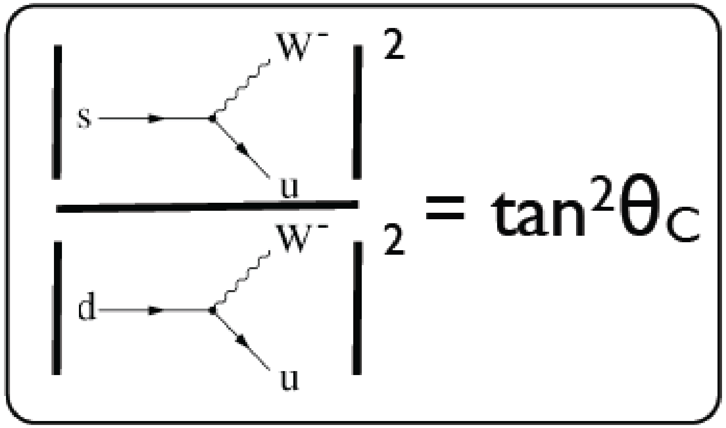
$$g \quad > \quad g\cos\theta_C \quad >> \quad g\sin\theta_C$$

To determine θ, let us compare the rates for $K^+ \rightarrow \mu^+ + \nu$ and $\pi^+ \rightarrow \mu^+ + \nu$; we find

$$\Gamma(K^+ \rightarrow \mu\nu)/\Gamma(\pi^+ \rightarrow \mu\nu)$$
$$= \tan^2\theta M_K(1 - M_\mu{}^2/M_K{}^2)^2/M_\pi(1 - M_\mu{}^2/M_\pi{}^2)^2. \quad (3)$$

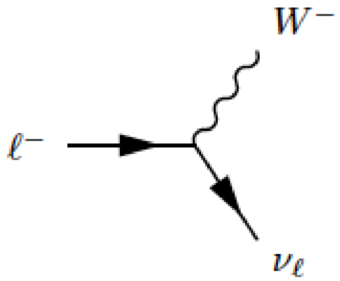
From the experimental data, we then get[5,6]

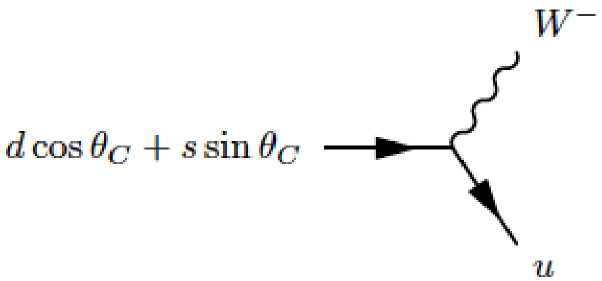
$$\theta = 0.257. \quad (4)$$



$$= \tan^2\theta_C$$

Universality of the couplings restaured !





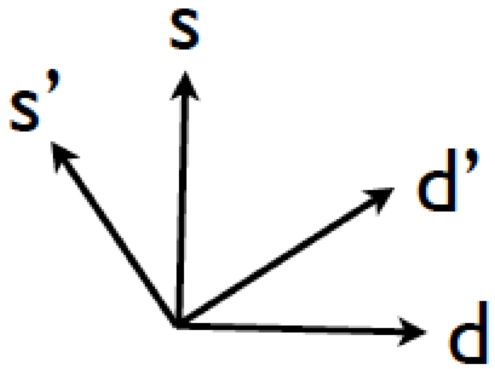
$$g \qquad = \qquad g$$

The d quark as ‘seen’ by the W, the *weak* eigenstate d’,
is *not* same as the *mass* eigenstate (the d)…

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} u \\ d' \end{pmatrix}_L = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}_L$$

d’ superposition of d and s

→ Existence of an s’ ?



$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

→ What about the up-type partner of the s’ (a c’?)

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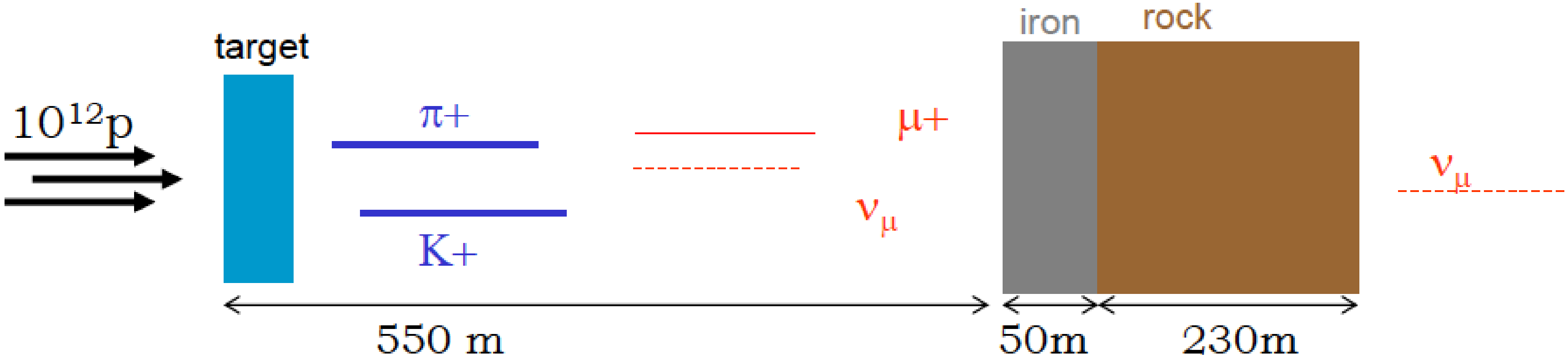
Neutral currents

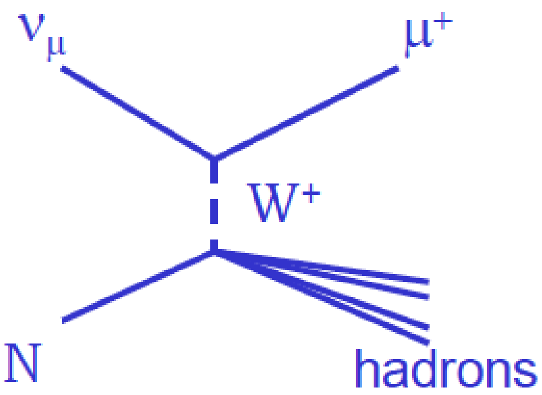
- Since the 60's : neutrinos beams can be set up

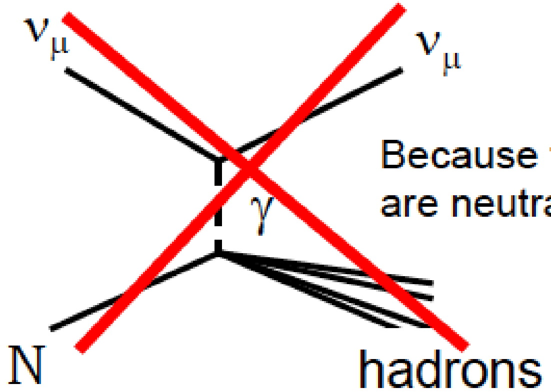


- In 1973 at CERN, the Gargamelle experiment (Hasert *et al*) discovers interactions of ν_μ without charged muon in the final state !

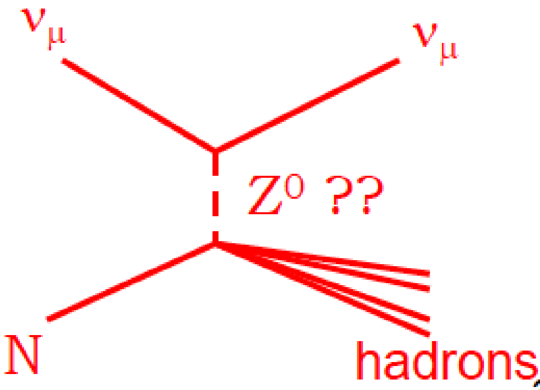
$\nu_\mu N \rightarrow \mu X$ charged current

$\nu_\mu N \rightarrow \nu_\mu X$ neutral current





Because the ν are neutral…

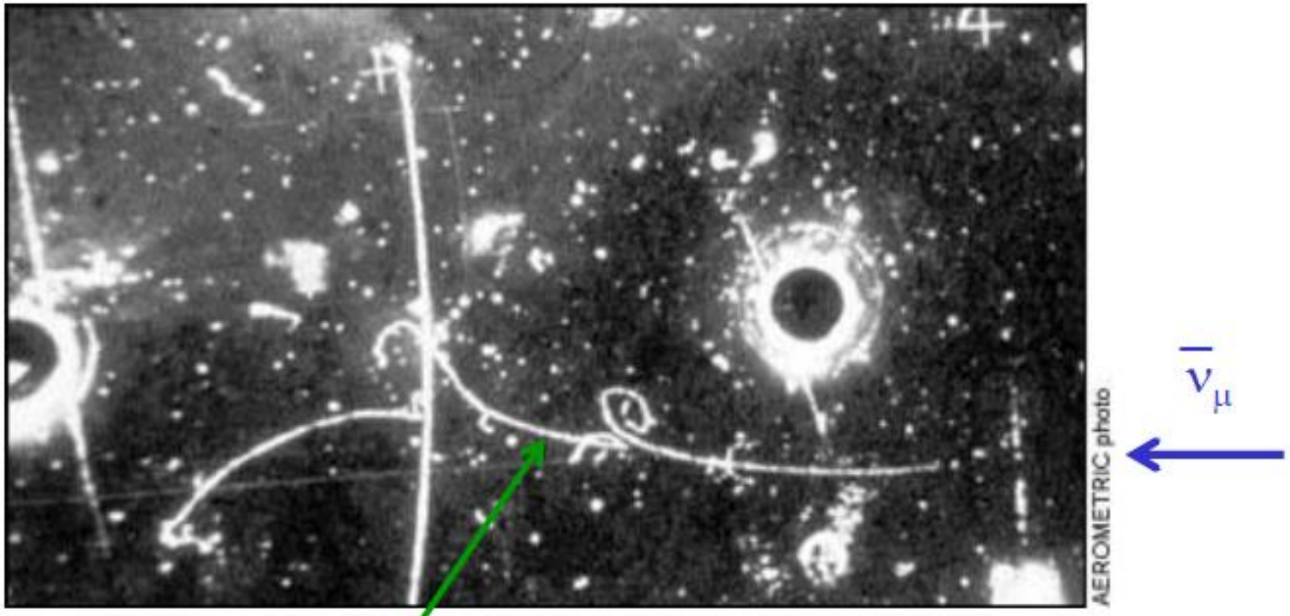


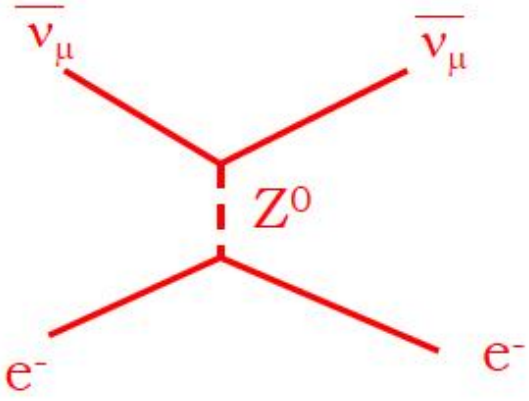
Gargamelle : Phys. Lett. B46, 138-140 (1973)

Discovery of the neutral currents :

Indirect evidence for a neutral vector boson mediating the weak interaction : the Z^0

First event : $\overline{\nu}_\mu e^- \to \overline{\nu}_\mu e^-$



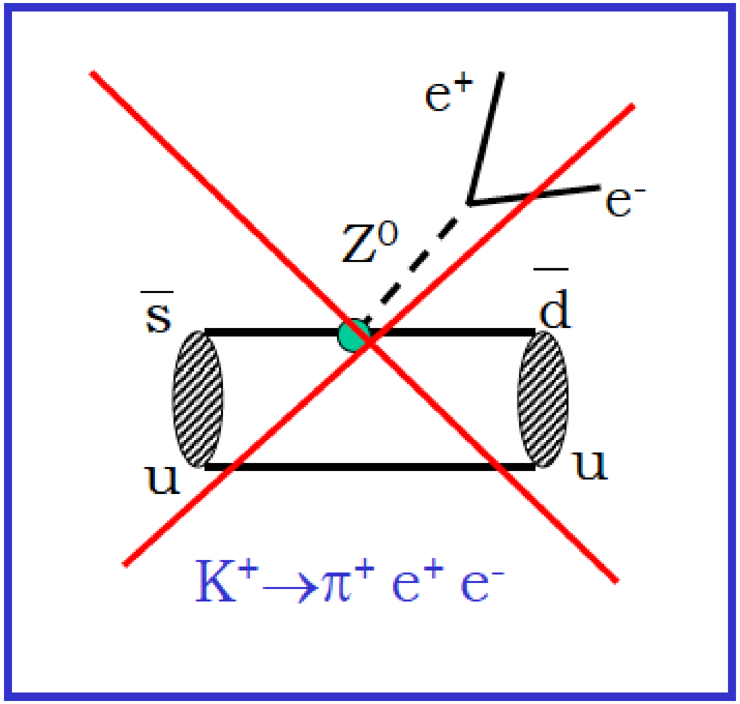


400 MeV e^- 400

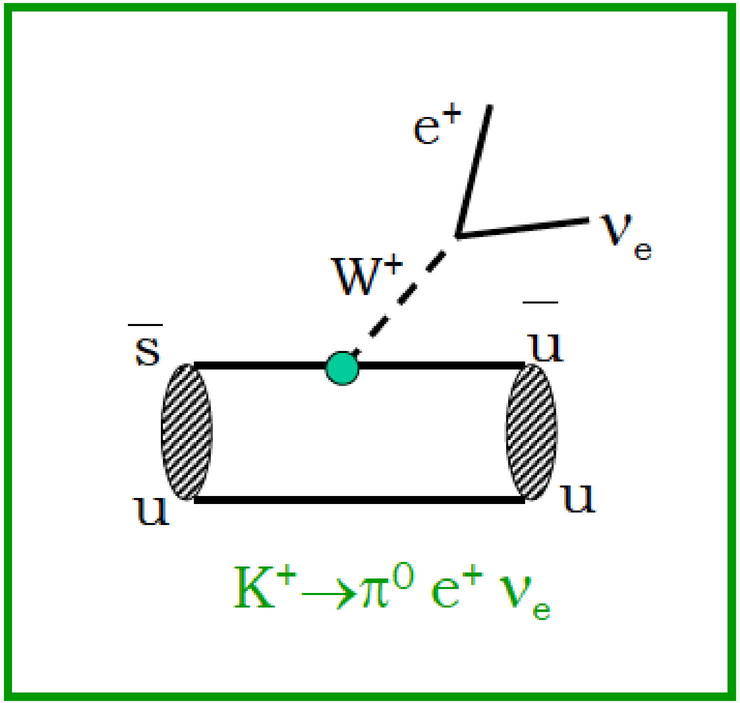
direction close from the direction of the incoming ν beam

Over a total of 1.4 million pictures:
3 events (data taking : 2 ans)

- The neutral currents are seen but no observation of :
  - $K^0 \rightarrow \mu^+ \mu^-$
  - $K^+ \rightarrow \pi^+ e^+ e^-$
  - $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



sd coupling



su coupling

No Flavour Changing Neutral Currents (FCNC) observed ($\Delta S=1$).

How to build a model explaining the properties of neutral currents ?

- Cabibbo's model :

$$\psi = \begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

Coupling: $$\bar{\psi}\psi = u\bar{u} + d\bar{d}\cos^2\theta_c + s\bar{s}\sin^2\theta_c + \left(s\bar{d} + d\bar{s}\right)\cos\theta_c\sin\theta_c$$

Neutral coupling predictions

The theory thus predicts the existence of neutral sd transitions (so $K^+ \to \pi^+ e^+ e^-$) which is incompatible with the observations.

- 1970 : Glashow, Iliopoulos et Maiani (GIM) propose the existence of a 4th quark: the c quark of charge 2/3 and thus of a new doublet :

$$\psi' = \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$

$$\bar{\psi}'\psi' = c\bar{c} + s\bar{s}\cos^2\theta_c + d\bar{d}\sin^2\theta_c - \left(s\bar{d} + d\bar{s}\right)\cos\theta_c\sin\theta_c$$

← To be added to the neutral coupling

→neutral coupling :

$$u\bar{u} + c\bar{c} + \left(d\bar{d} + s\bar{s}\right)\cos^2\theta_c + \left(d\bar{d} + s\bar{s}\right)\sin^2\theta_c = u\bar{u} + c\bar{c} + d\bar{d} + s\bar{s}$$

The GIM mechanism allows to take into account the non observation of Flavour Changing Neutral (ΔS=0,ΔC=0).

$$(\bar{u},\bar{c})\gamma^{\mu}(1-\gamma_5)V\begin{pmatrix} d \\ s \end{pmatrix}$$

1° Prediction

a fourth quark

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = V\begin{pmatrix} d \\ s \end{pmatrix} \qquad V = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

neutron decay $\quad u\bar{d} \quad \sim G_F^2\cos^2\theta_c \sim G_F^2$

Strange particles $\quad u\bar{s} \quad \sim G_F^2\sin^2\theta_c$

2° Prediction

Decay couplings of the fourth quark

Charm sector $\quad \begin{cases} c\bar{d} & \sim G_F^2\sin^2\theta_c \\ c\bar{s} & \sim G_F^2\cos^2\theta_c \sim G_F^2 \end{cases}$

The charm discovery in 1974 and the verification of these predictions have been a tremendous triumph of this picture and these predictions have been verified : c→d are Cabibbo suppressed wrt c→ s transitions

Back-up slides

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Weak interaction the V-A structure

Projection of the spin in the momentum direction

4.1 Helicity/Chirality :

- Helicity: definition for the helicity operator : $H = \dfrac{\sigma . p}{|\vec{p}|}$ with $\sigma = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$
- Chirality:

  If ψ is a solution for the Dirac equation, on can write: $\psi = \psi_{CL} + \psi_{CR}$

  Definition: chirality operators *Left ou Right* (CL,CR) :

$$\boxed{\begin{aligned} P_{CL} &= \frac{1-\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{1} \end{pmatrix} \\ P_{CR} &= \frac{1+\gamma_5}{2} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}}$$

$$\gamma_5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \quad 4 \times 4$$

(for the chiral representation)

Algebra :

$$P_{CL}^2 = P_{CL}, \quad P_{CR}^2 = P_{CR}, \quad P_{CL} + P_{CR} = 1, \quad P_{CL} P_{CR} = 0$$

$$\psi_{CL} = P_{CL}\psi \qquad \overline{\psi}_{CL} = \overline{\psi} P_{CR}$$

$$\psi_{CR} = P_{CR}\psi \qquad \overline{\psi}_{CR} = \overline{\psi} P_{CL}$$

$$P_{CL}\gamma^{\mu} = \gamma^{\mu} P_{CR} \qquad P_{CR}\gamma^{\mu} = \gamma^{\mu} P_{CL}$$

Link between helicity and chirality

- Chirality is the "correct" quantity (it appears in the Lagrangian and in addition the helicity in not Lorentz invariant) but what is measured is the helicity and this is also the helicity which is preserved in the reactions !

- One can show that:

$$\psi_{CL} = \frac{a}{2}\psi_{HR} + \frac{b}{2}\psi_{HL} \qquad \text{with} \qquad a = 1 - \frac{p}{E+m}$$

$$\psi_{CR} = \frac{b}{2}\psi_{HR} + \frac{a}{2}\psi_{HL} \qquad\qquad b = 1 + \frac{p}{E+m}$$

ψ_{CR}, ψ_{CL} are the eigenvectors of H

ψ_{CR} corresponds to the eigenvalue +1

ψ_{CL} corresponds to the eigenvalue -1

- for m<<E: a = 1 - β and b = 1 + β

  $\beta \sim 1$: a = 0 and b = 2

  and thus : $\psi_{CL}=\psi_{HL}$ and $\psi_{CR}=\psi_{HR}$

for E>>m : Helicity $\equiv$ Chirality

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4.2 V-A structure:

Let's take an electromagnetic current : $\bar{\psi}\gamma^\mu\psi$:

(γ^μ is a vector, parity = -1)

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}\left(P_{CL}+P_{CR}\right)\gamma^\mu\left(P_{CL}+P_{CR}\right)\psi =$$

$$\bar{\psi}\cancel{P_{CL}\gamma^\mu P_{CL}}\psi + \bar{\psi}P_{CR}\gamma^\mu P_{CL}\psi + \bar{\psi}P_{CL}\gamma^\mu P_{CR}\psi + \bar{\psi}\cancel{P_{CR}\gamma^\mu P_{CR}}\psi =$$

$$\bar{\psi}_{CL}\gamma^\mu\psi_{CL} + \bar{\psi}_{CR}\gamma^\mu\psi_{CR}$$

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_{CL}\gamma^\mu\psi_{CL} + \bar{\psi}_{CR}\gamma^\mu\psi_{CR}$$ selects ψ_{CL}, ψ_{CR}

$\Rightarrow$ for the electromagnetic interaction : ψ_{CL} - ψ_{CL} and ψ_{CR} - ψ_{CR} couplings

Let's take a weak coupling $\bar{\psi}\gamma^\mu(1-\gamma_5)\psi$:

$\gamma^\mu\gamma^5$ is vector-axial, parity +1

$\gamma^\mu(1-\gamma_5)$: V-A structure

$$\bar{\psi}\gamma^\mu\left(1-\gamma_5\right)\psi = \bar{\psi}\left(P_{CL}+P_{CR}\right)\gamma^\mu\left(1-\gamma_5\right)\left(P_{CL}+P_{CR}\right)\psi =$$

$$2\bar{\psi}\left(P_{CL}+P_{CR}\right)\gamma^\mu\left(P_{CL}^2+\cancel{P_{CL}P_{CR}}\right)\psi =$$

$$2\bar{\psi}P_{CL}\gamma^\mu P_{CL}\psi + 2\bar{\psi}P_{CR}\gamma^\mu P_{CL}\psi = 2\bar{\psi}\gamma^\mu\cancel{P_{CR}P_{CL}}\psi + 2\bar{\psi}_{CL}\gamma^\mu\psi_{CL}$$

$$\bar{\psi}\gamma^\mu\left(1-\gamma_5\right)\psi = 2\bar{\psi}_{CL}\gamma^\mu\psi_{CL}$$

$\Rightarrow$ for the weak interaction : ψ_{CL} - ψ_{CL} coupling only

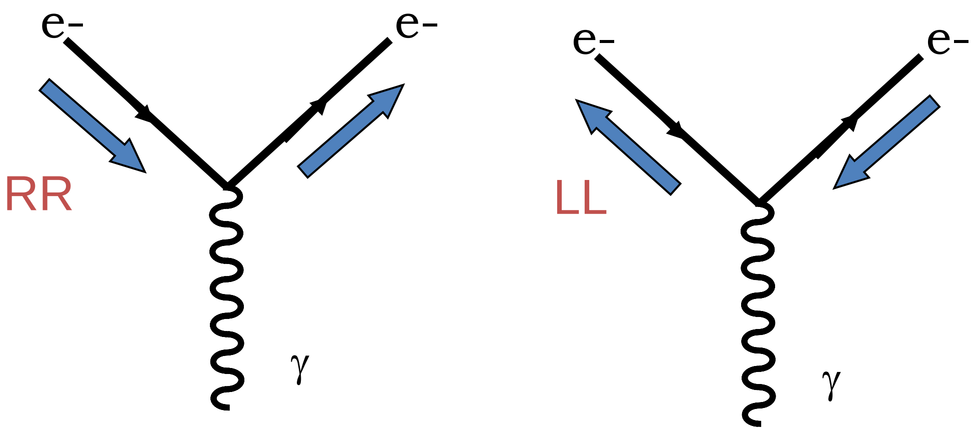
This form for the current leads to maximal parity violation
(the V-A structure allows only left handed neutrinos)

Validation of the $\gamma^\mu(1-\gamma_5)$ expression for the weak currents

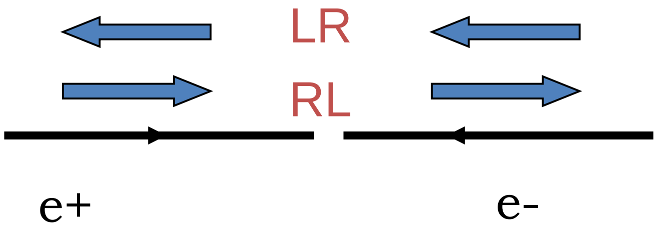
In the limit E>>m : Helicity = Chirality

Electromagnetic interaction

particle-particle : LL or RR

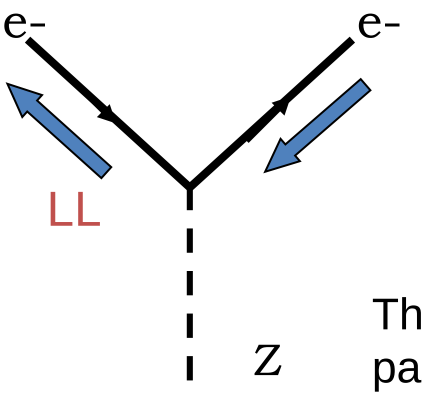


particle-antiparticle : LR or RL

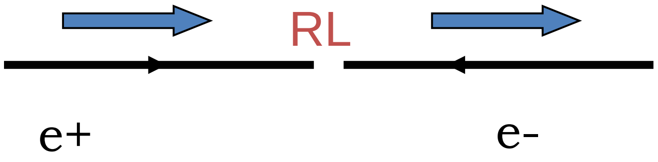


Weak interaction

particle-particle : LL



particle-antiparticle : LR



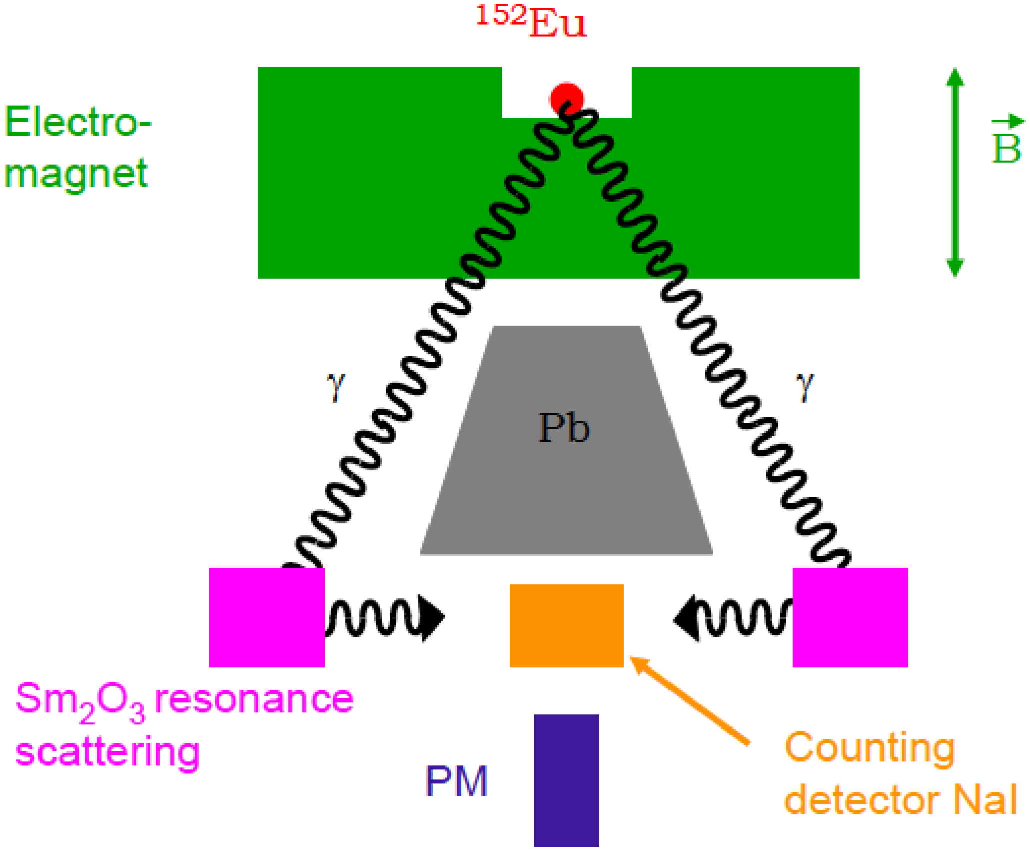
The weak interaction selects Left handed particles and Right handed anti-particles

Experimental evidence for the neutrinos helicity:

Goldhaber *et al*
Phys. Rev. 109, 1015-1017
(1958)

Reminder: the helicity is given by the projection of the spin on the particle momentum

Experimental set-up:

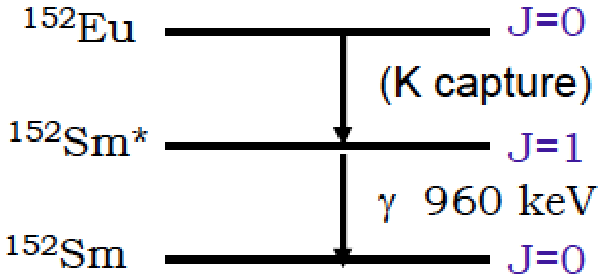


Studied decay:

$$^{152}\mathrm{Eu}(J=0) + e^- \rightarrow {}^{152}\mathrm{Sm}^*(J=1) + \nu$$

(K capture)

$$\hookrightarrow {}^{152}\mathrm{Sm}(J=0) + \gamma$$

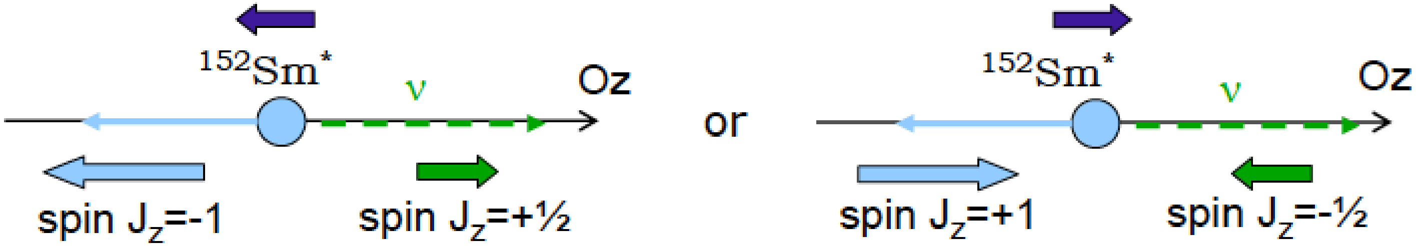


γ emitted in the direction of the momentum of the Sm* are selected

- $^{152}\mathrm{Eu}(J=0) + e^{-} \rightarrow {}^{152}\mathrm{Sm}^{*}(J=1) + \nu$

initial state: $J_i = ½ \Rightarrow J_f = ½$, J_z given by the **electron**

In order to get $J_f = ½$, the projection of the spin of the Sm* and of the ν should be opposite (same helicities)



$J_z(\mathrm{Sm}^*)=0$ forbidden by the decay mode

- decay : $^{152}\mathrm{Sm}^*(J=1) \rightarrow {}^{152}\mathrm{Sm}(J=0) + \gamma$
  J_z=+/-1 J_z=0 J_z=+/-1 (forbidden $J_z(\mathrm{Sm}^*) = 0$)

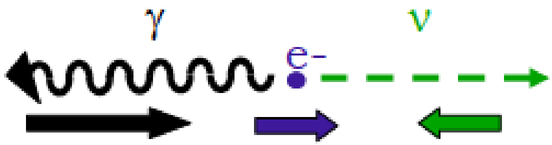
γ emitted forward in the ^{152}Sm direction are selected
the 3 final state particles (Sm, γ and ν) are collinear.

- helicities of the final state particles : S(ν)=± ½ , S(γ)= ±1, S(e)= ± ½

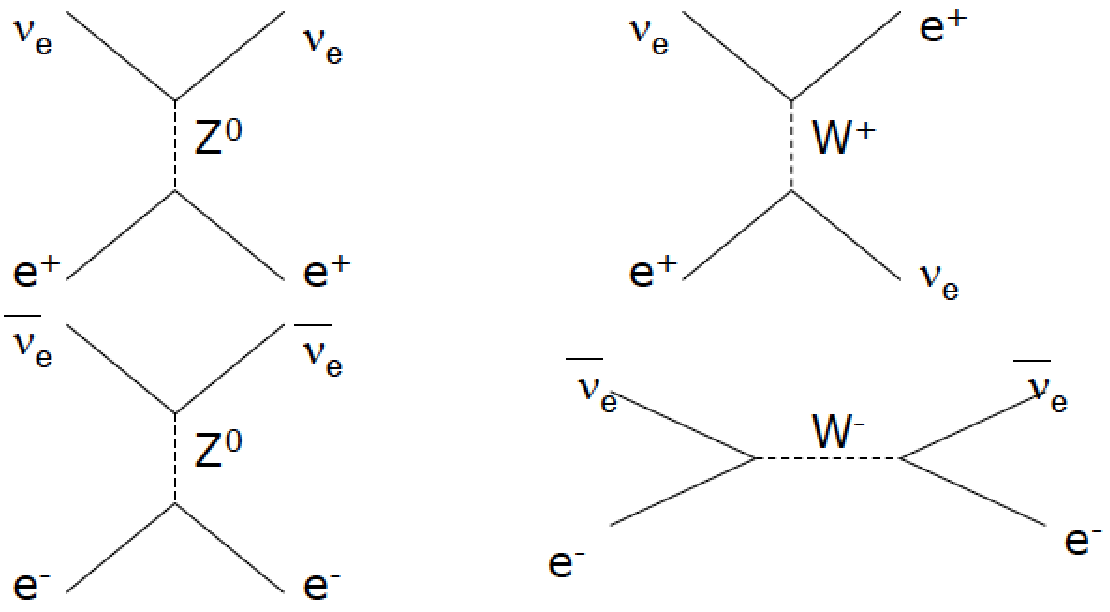
Two possible configurations :

⇒ The γ and ν helicities are the same.

- The γ polarization is measured to measure the neutrinos helicity. One sees only left handed neutrinos :



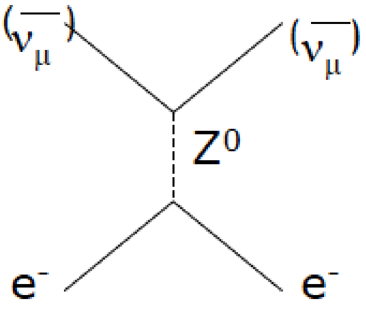
Why is $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ an unambiguous (but still indirect !) sign of the existence of the Z^0 boson ?



$\nu_e e^+ \to \nu_e e^+$

2 diagrams with W and Z

$\bar{\nu}_e e^- \to \bar{\nu}_e e^-$



$\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$

No diagram with a W !
(would violate the leptonic number)