Introduction to Particle Physics

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- Introduction to Particle Physics (history/interaction/symmetries)
- The strong interaction
- The weak interaction
- Heavy Flavours
- The Standard Model and Higgs
- Heavy Flavours to search for New Physics
- Few open questions

+ EXCERCISES !

Chapter I

Introduction to Particle Physics

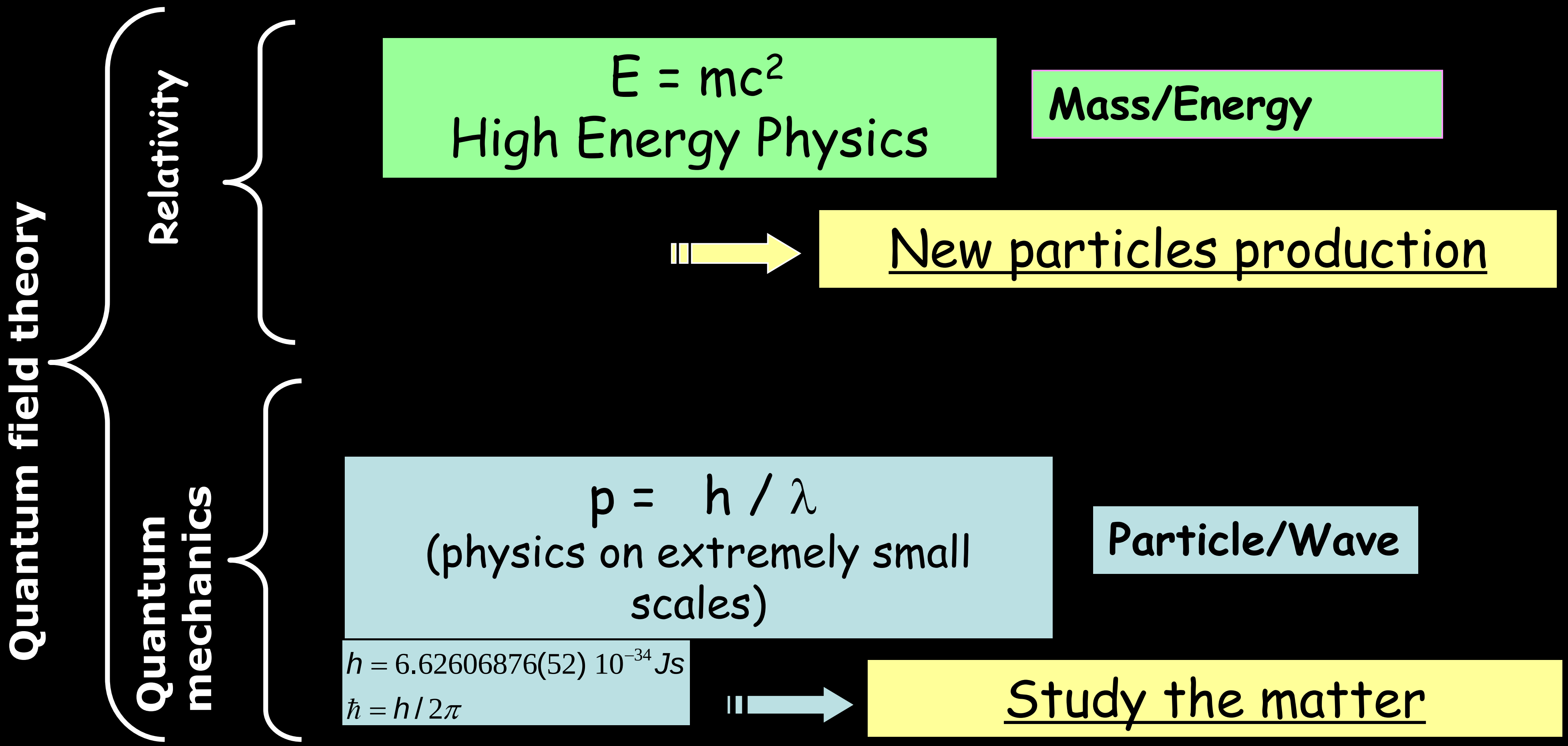
I.1

What is particle physics about ?

The particle world

The laws of « this world » are not really intuitive..

$e = 1.602176462(63)\ 10^{-19}\ C$
$m = 9.10938188(72)\ 10^{-31}\ \text{kg}$



Quantum field theory

Relativity

$E = mc^2$
High Energy Physics

Mass/Energy

New particles production

Quantum mechanics

$p = h / \lambda$
(physics on extremely small scales)

$h = 6.62606876(52)\ 10^{-34}\ Js$
$\hbar = h / 2\pi$

Particle/Wave

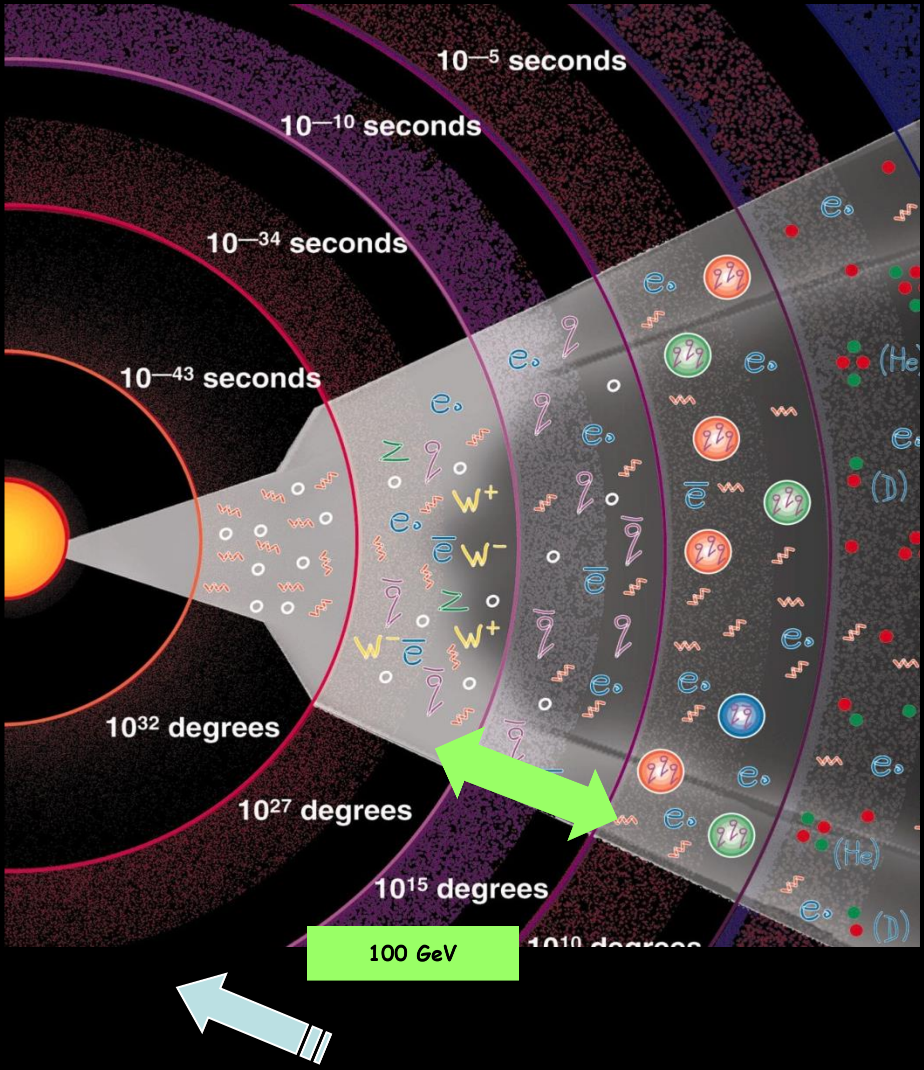
Study the matter

Particle world is described by quantum field theory

It is our main working tool for particles physics

It comes from the marriage between quantum mechanics and relativity

The particle world : Physics of the two-infinities



Produce particles
at 100GeV ~ 10^{-8} Joule

↓

Temperature ~ 10^{15} degrees

↓

Condition of the Universe
after ~10^{-10} sec from Big Bang

<u>Particles (which are very small « objects ») of high energy are instruments to go back in time (very large scales)</u>

The mass

Defined by : $m^2c^4=E^2-p^2c^2$ ← Invariant length of the Energy-momentum 4-vector

With $c=1$ E, p and m are expressed using the same unity (GeV/MeV ….)

- When $p=0$ $\Rightarrow$ $E = mc^2$
- When v increases $\Rightarrow$ E^2 et p^2c^2 increase but their difference remains constant
- m is a Lorentz invariant

New particles production:

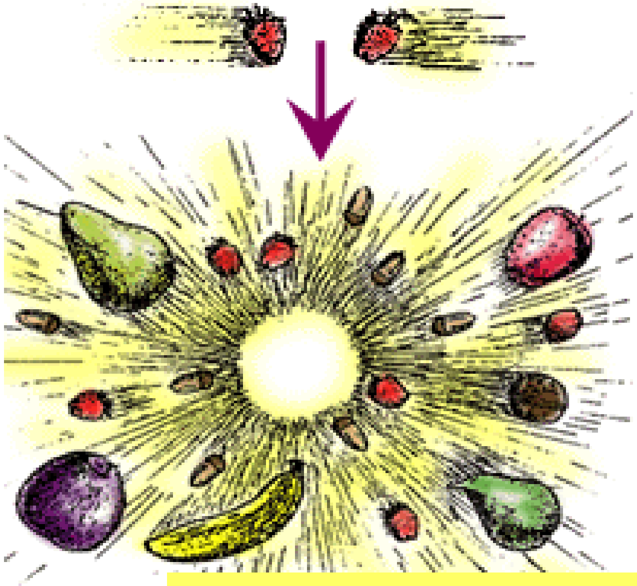
It is not "divisibility" !

Since c is large

small mass

=

Large energy



A particle is a lump of energy

Mass/energy

mass
energy

energy
mass

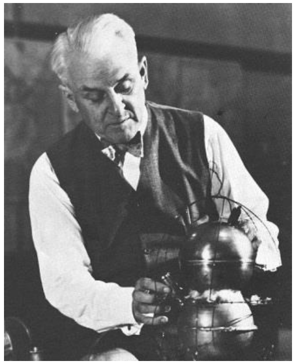
MICROSCOPIC WORLD

Thompson experiment

Determination of m/e for electrons

Determination of the quantum nature and the value of the electric charge for electrons

Millikan experiment



Today

- $e = 1.602176462(63)\ 10^{-19}$ C
- $m = 9.10938188(72)\ 10^{-31}$ kg

1 Joule =1Coulomb*1 Volt

1eV = Energy for an electron fealing a potential difference of 1 V

1eV = 1.6 10^{-19} Joule

$mc^2 = 9.1\ 10^{-31}\ \text{kg} \times (3\ 10^8)^2\ \text{m}^2/\text{sec}^2 = 50\ 10^4\ \text{eV}$

$1\text{eV}/c^2 = 1.78\ 10^{-36}$ kg

$m_e = 0.5\ \text{MeV}/c^2 = 0.5$ MeV (c=1)

$m_p = 938\ \text{MeV} \approx 1$ GeV

KeV (10^3 eV)

MeV (10^6 eV)

GeV(10^9 eV)

TeV (10^{12} eV)

I.2

An historical introduction

Historical overview

A bit of history

- **1897** (*Thompson)* electron discovery
- **1912** (*Rutherford*) proton discovery
- **~1930** (*Pauli/Fermi*) neutrino ν_e hypothesis

  1958 (*Reines-Cowan)* : experimental evidence
- **1932** (*Chadwick*) neutron discovery
- **1932** (*Anderson*) positron discovery :e^+ 1st antiparticle

In the '30s one knows : e^- p n and ν_e the electromagnetic force and the γ

Framework to try to explain the forces between p, then between p and n ⇒ Strong interaction

Observation of unstable particles in cosmic rays + β decays ⇒ Weak interaction

Many particles :

Increasing mass

- •1937 $\mu \rightarrow e\nu_e\nu_\mu$ — Discovered in 1962
- •1947 $\pi \rightarrow \mu\nu_\mu$
- •1947 strange particles : K Λ

When the muon was discovered the physicist I. Rabi said :

It remains in fact a very good question…..

- **1955** (*Chamberlain, Segre, Wiegand, Ypsilantis*) antiproton discovery

1950

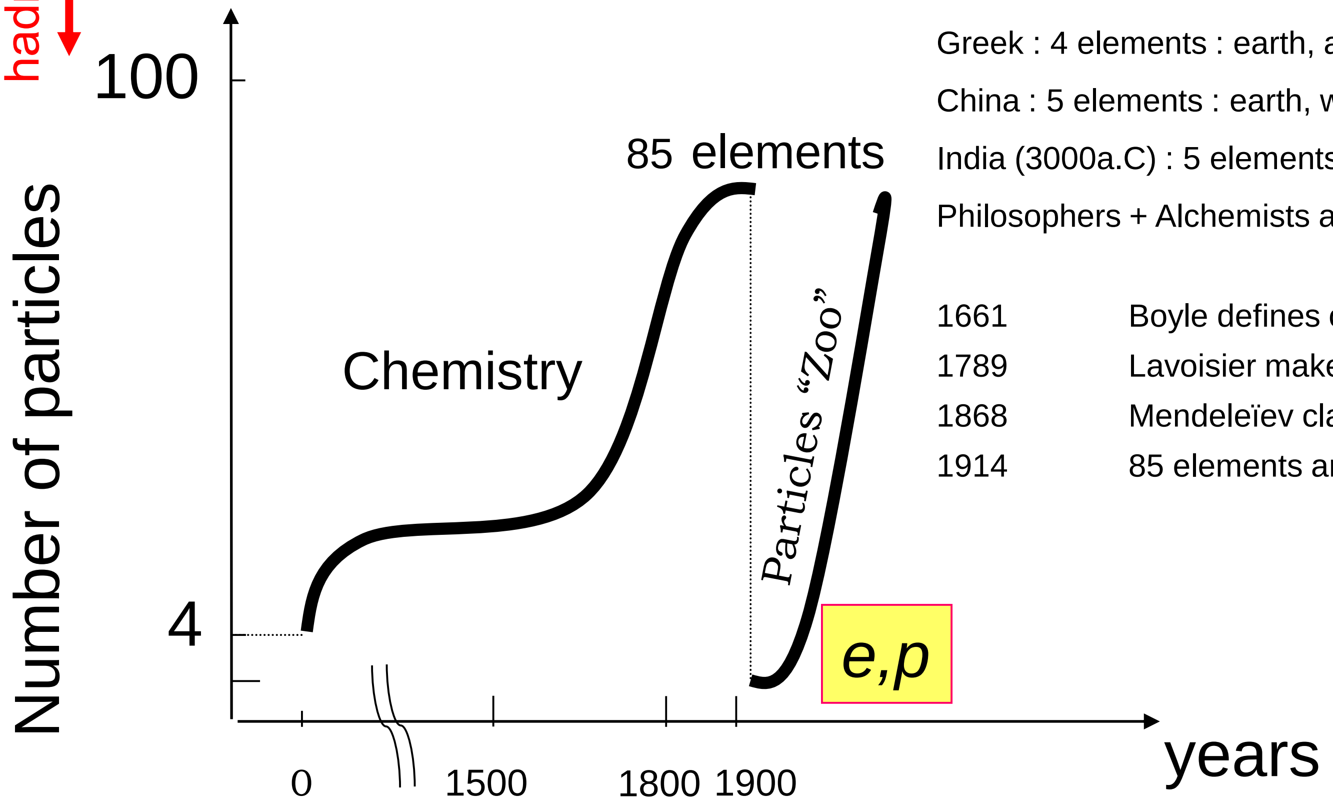
Accelerators era :

Larger statistics

Control on the particles

More and more hadrons …

- •1948 first π produced
- •1950 π^0 discovery
- •1954 K^+ K^0 , Σ production …
- •1964 Ω discovery



Greek : 4 elements : earth, air, fire, water

China : 5 elements : earth, wood, metal, fire and water

India (3000a.C) : 5 elements :space, air, fire, water and

Philosophers + Alchemists add : ether, mercury, sulphur, salt

1661	Boyle defines chemistry
1789	Lavoisier makes the list of 33 elements
1868	Mendeleïev classify the elements
1914	85 elements are known

1964 *Zweig-GellMann-Neeman* : theoretical introduction of quarks

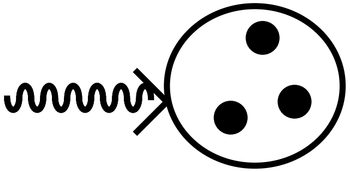
'70 SLAC

Deep inelastic scattering experiments

Experimental evidence of quarks

p≡(uud) n ≡(udd)

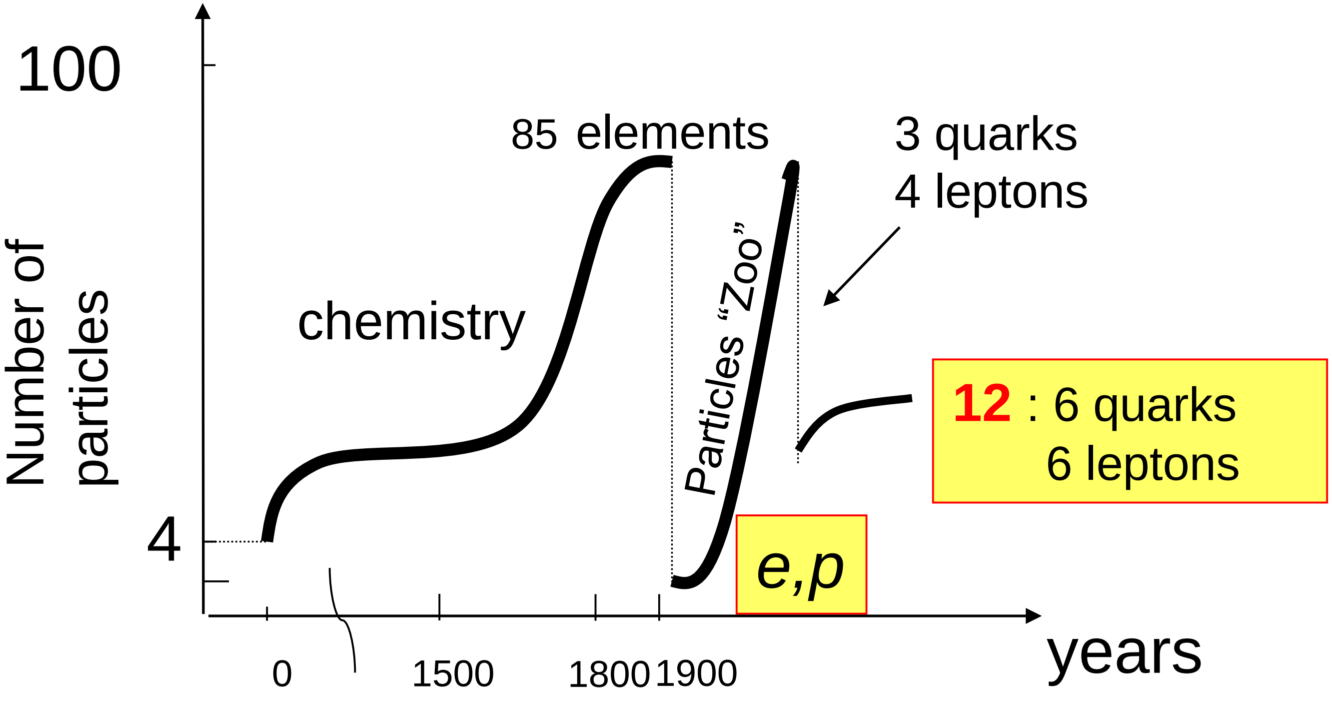
Strange particles (Λ,K…): quark s

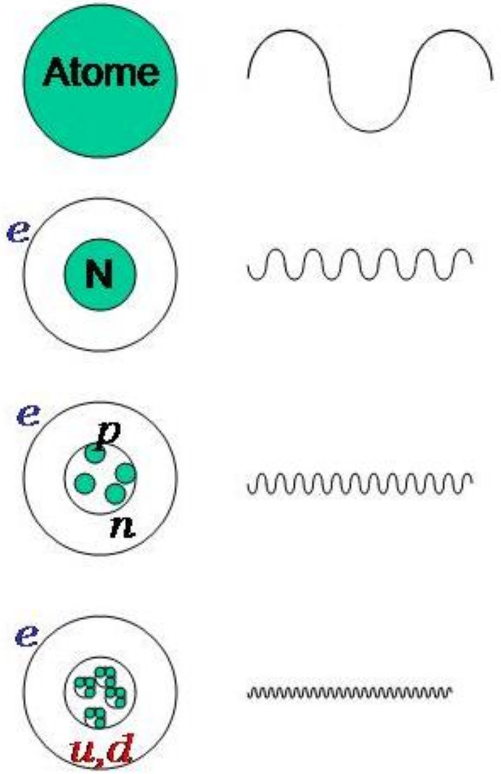


'90 HERA

…. New particles..

- 1974 (*Richter/Ting*) J/ψ discovery ($c\bar{c}$) : c quark
- 1975 (*Perl*) discovery of the τ lepton
- 1977 (*Ledermann*) Υ discovery ($b\bar{b}$ bound state) : b quark
- 1995 (*CDF/DØ coll.*) t quark
- 2000 (Donut) : ν_τ

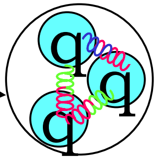




12 matter particles to explain all known particles !

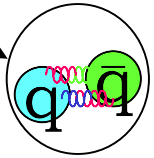
Hadrons : any particle which undergoes the strong interaction

(Nucleon : neutron and proton)



Baryons half integer spin

(ex: p =(uud))



Mesons integer spin

(ex: π = $(u\bar{d})$)

Leptons : Any particle which does not undergo the strong interaction (e,μ,τ) $(\nu_e,\nu_\upsilon,\nu_\tau)$

Interaction particles

- Classical mechanics : interactions = force field
- Modern physics : interactions = interaction particles which are field quanta

1973 Observation at CERN of the "weak neutral currents"
Interactions between neutrinos → "Z^0" ?

1976 Standard Model. Electroweak unification
$\Rightarrow \gamma, Z^0\ W^{+-}$ → They are vectors of the weak interaction, their masss are predicted

1983 Observation at CERN of the Z^0 and W^{+-} bosons

1989 « Mass production» of Z^0 at LEP at CERN

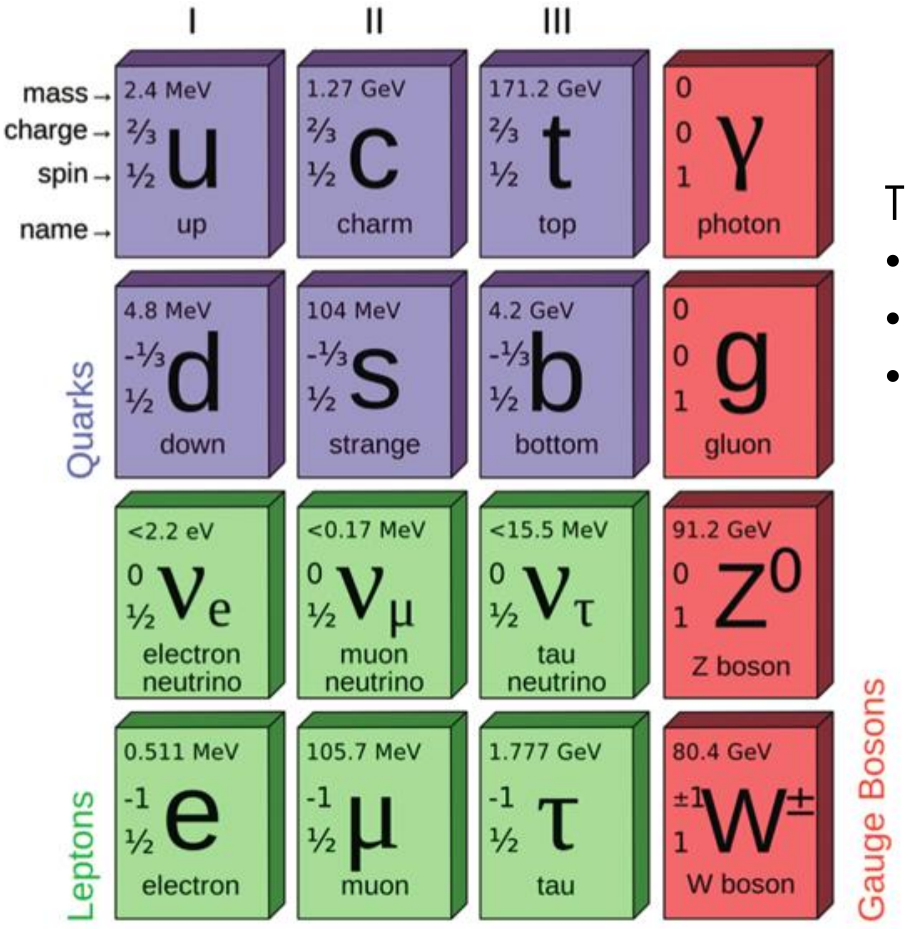
1996 Production of W^+W^- pairs at LEP at CERN

2012 DISCOVERY OF THE HIGGS BOSON AT LHC (CERN)

Elementary particles

3 families of fermions : matter **+ anti-matter !**

3 forces : electromagnetism, weak interaction, strong interaction



	I	II	III	
mass → charge → spin → name →	2.4 MeV ⅔ ½ u up	1.27 GeV ⅔ ½ c charm	171.2 GeV ⅔ ½ t top	0 0 1 γ photon
Quarks	4.8 MeV -⅓ ½ d down	104 MeV -⅓ ½ s strange	4.2 GeV -⅓ ½ b bottom	0 0 1 g gluon
	<2.2 eV 0 ½ ν_e electron neutrino	<0.17 MeV 0 ½ ν_μ muon neutrino	<15.5 MeV 0 ½ ν_τ tau neutrino	91.2 GeV 0 1 Z^0 Z boson
Leptons	0.511 MeV -1 ½ e electron	105.7 MeV -1 ½ μ muon	1.777 GeV -1 ½ τ tau	80.4 GeV ±1 1 $W^\pm$ W boson

Gauge Bosons

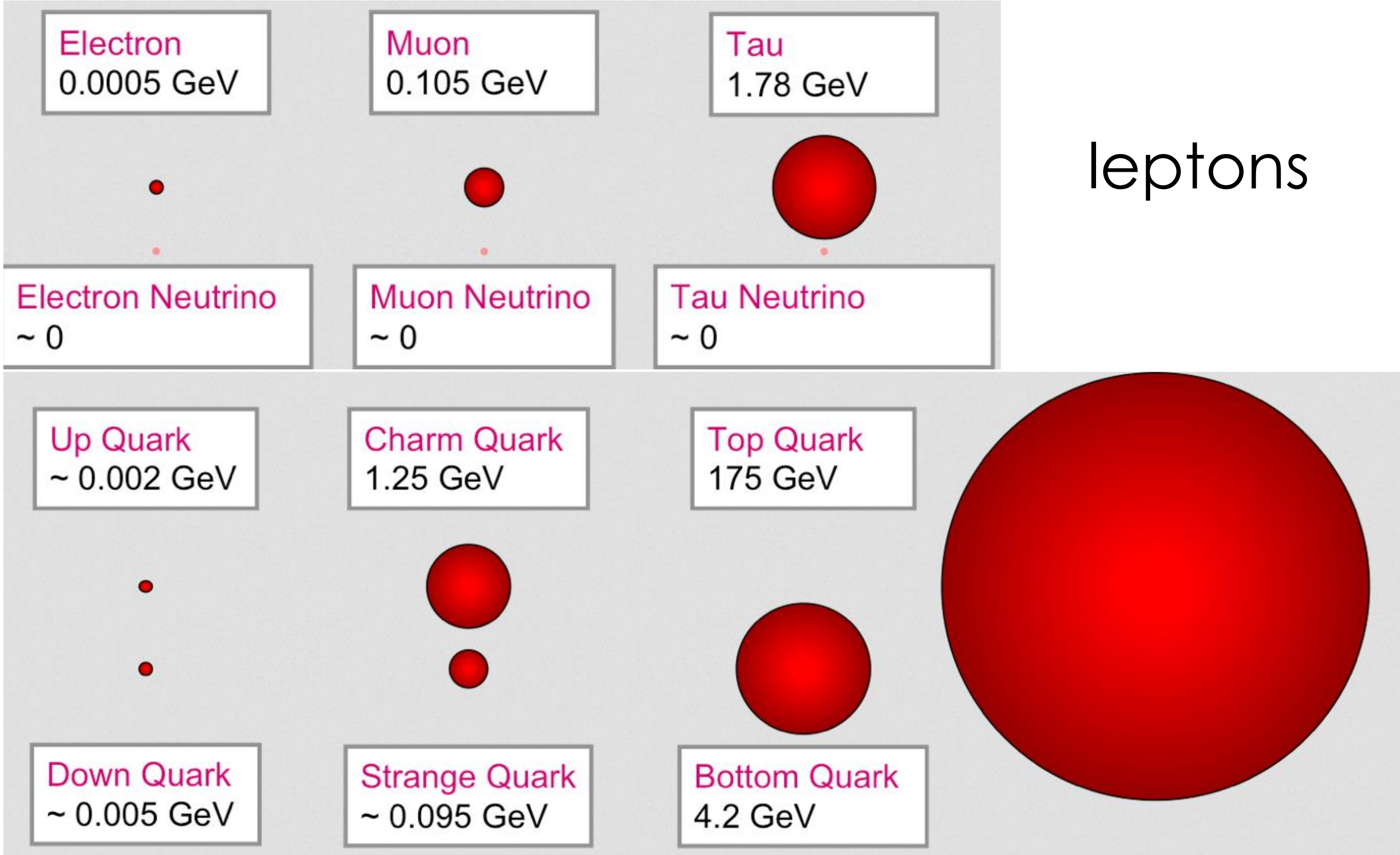
And the Higgs boson !

The particles are characterized by :

- their spin
- their mass
- the quantum numbers (charges) determining their interactions

All our knowledge is today « codified » in the **Standard Model** :
Matter, Interaction, Unification Interaction, Unification

The fermions and their masses



Electron
0.0005 GeV

Muon
0.105 GeV

Tau
1.78 GeV

Electron Neutrino
~ 0

Muon Neutrino
~ 0

Tau Neutrino
~ 0

leptons

Up Quark
~ 0.002 GeV

Charm Quark
1.25 GeV

Top Quark
175 GeV

Down Quark
~ 0.005 GeV

Strange Quark
~ 0.095 GeV

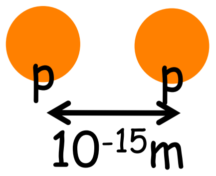
Bottom Quark
4.2 GeV

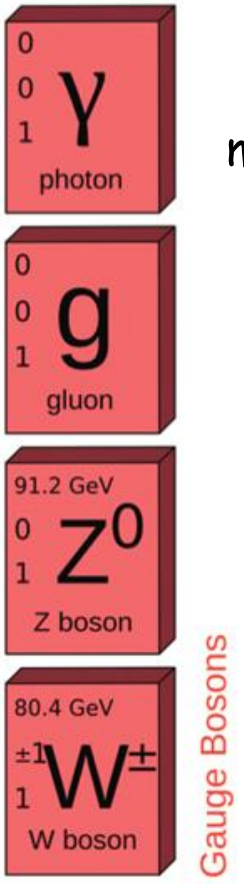
quarks

1st family 2nd family 3rd family

The interactions and their mediators

Spin 1 particles





m=0	Electromagnetism	10^{-2}
m=0	Strong interaction	1
m=91.2 GeV	Weak interaction	10^{-8}
M=80.4 GeV		

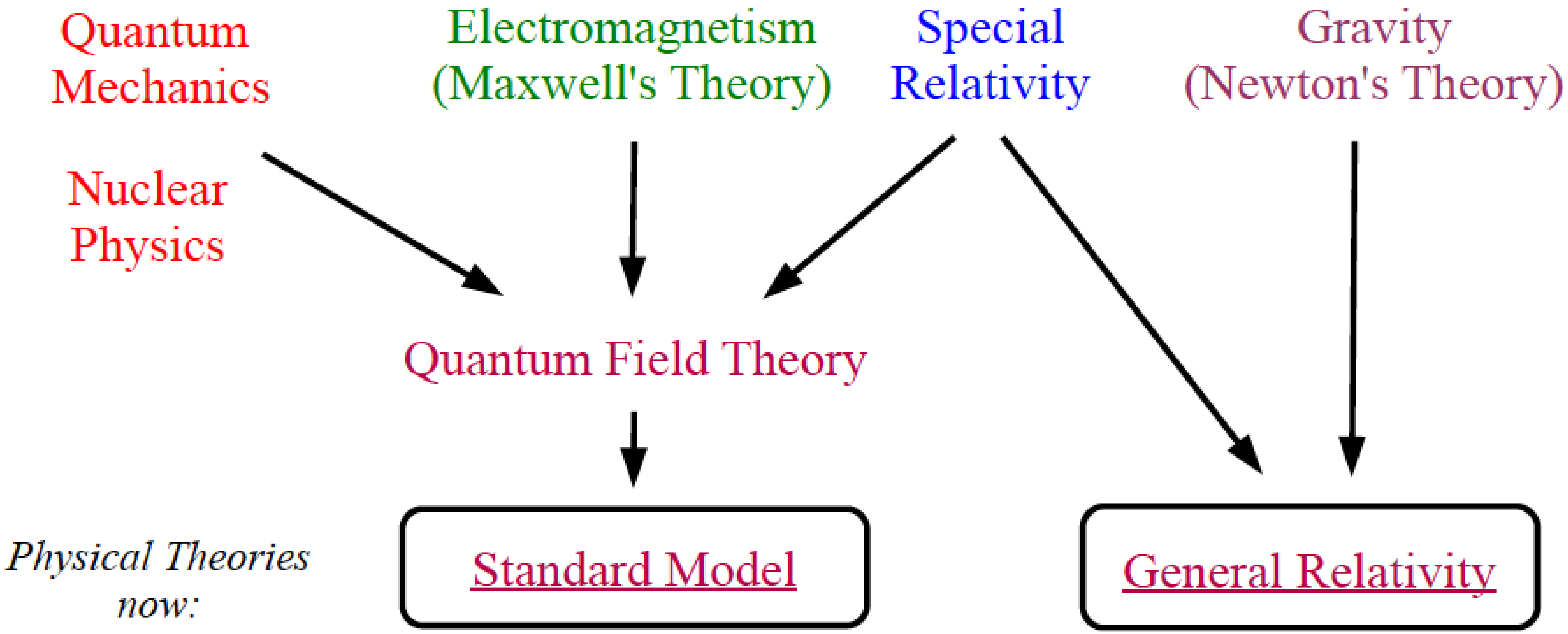
Gravity :
negligible at the scale of elementary particles
We do not know today how to quantify it

Probe the underlying structure of matter

Production of new particles

$p = h/\lambda$
(towards the smallest scales)

$E = Mc^2$
(High energy physics



Quantum Mechanics

Nuclear Physics

Electromagnetism (Maxwell's Theory)

Special Relativity

Gravity (Newton's Theory)

Quantum Field Theory

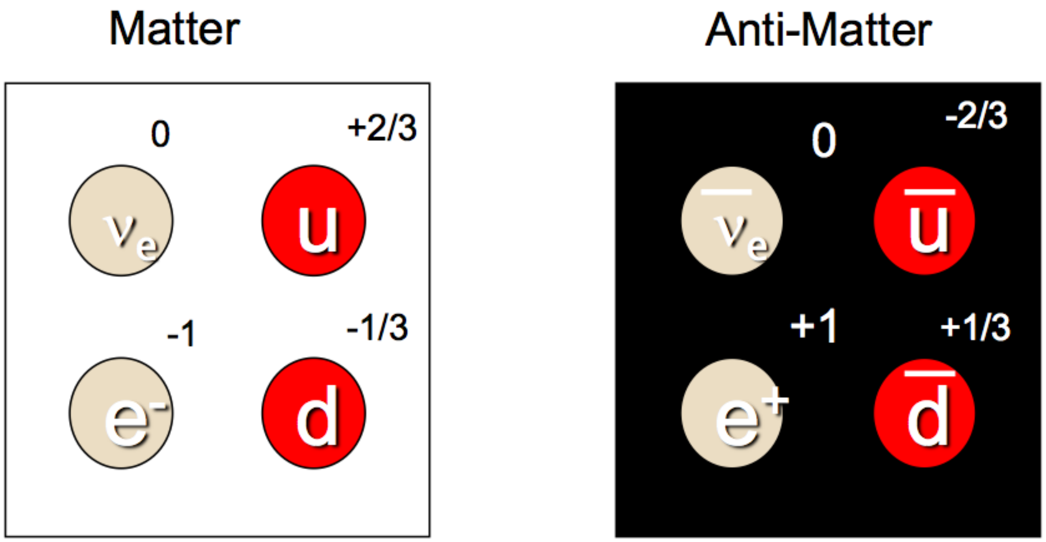
Physical Theories now:

Standard Model

General Relativity

Anti-matter ?

To each particle one can associate an anti-particle : same mass but all quantum numbers opposite



In 1931 Dirac predicts the existence of a particle similar to the electron but of charge +e

Discovery of the positron

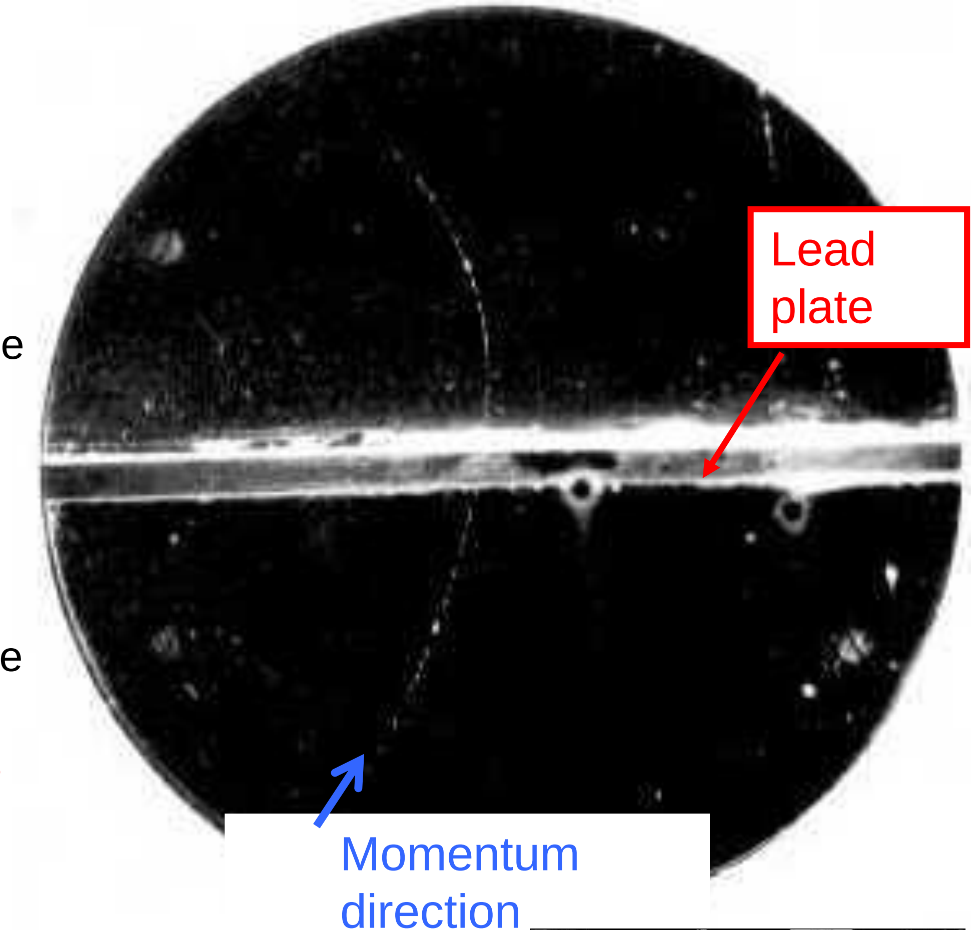
- The radius of curvature is smaller above the plate. The particle is slow down in the lead

  → the particle in incoming from the bottom
- The magnetic field direction is known

  → positive charge
- From the density of the drops one can measure the ionizing power of the particle → minimum ionizing particle
- Similar ionizing power before and after the plate

  → same particle on the 2 sides
- Curvature measurement after the lead : particle of ~23MeV

  → it is not a non-relativistic proton because he would have lost all its energy after ~5mm (a track of ~5 cm is observed)

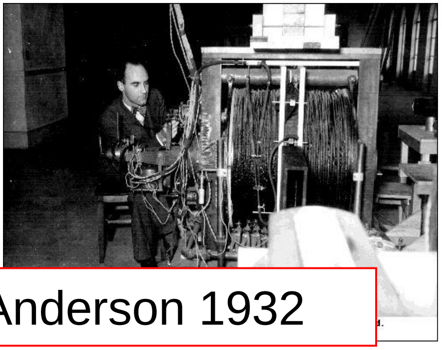


Lead plate

Momentum direction

Particle of positive electric charge and with a mass

much smaller than the proton mass (< 20 m_e) : the **positron**



Anderson 1932

I.3

Two important observables :

Lifetime/Width : τ / Γ

Cross Section : σ

Lifetime : τ

Lifetime : the exponential law

Instable particles and nuclei : number of decays per unit of time ($\Delta N/\Delta T$) proportional to the number of particles/nuclei (N)

$\Delta N = cte \times N \times \Delta t \Rightarrow$ exponential law

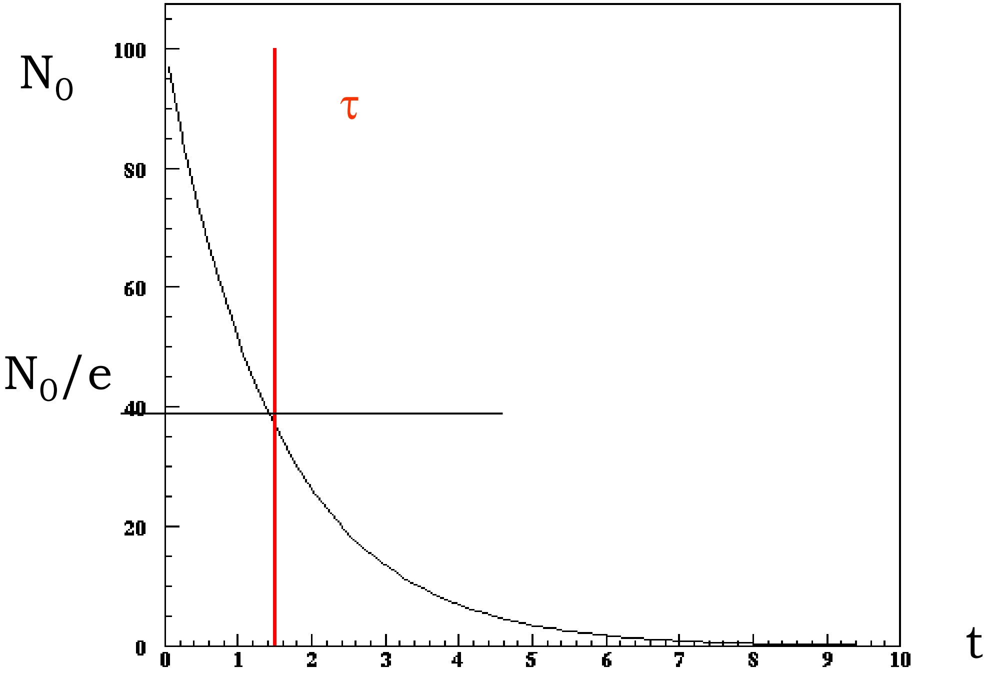
$N(t) = N_0 e^{-t/\tau}$

Mean lifetime (defined in the particle rest frame)

The majority of the particles are instable

τ from 10^{-23} sec (resonances)

to ~10^{+3} sec (neutron)



The probability for a radioactive nucleus to decay during a time interval *t*, does not depend on the fact that the nucleus has just been produced or exists since a time T :

$$\left[\begin{array}{l}\textbf{Survival probability}\\ \textbf{after the time T} + \textbf{t}\end{array}\right] = \left[\begin{array}{l}\textbf{Survival probability}\\ \textbf{after the time T}\end{array}\right] \times \left[\begin{array}{l}\textbf{Survival probability}\\ \textbf{after the time t}\end{array}\right]$$

$e^{a+b} = e^a \times e^b$

Few important examples of different lifetimes

- Stable particles : γ,e,p,ν → the only ones !

proton stability $\tau(p) > \sim 10^{32}$ ans

- particles with long lifetimes :

$n \rightarrow p + e^- + \overline{\nu}_e$	$\tau = 6.13\ 10^{+2}$ sec, β decay
$\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$	$\tau = 2.2\ \ 10^{-6}$ sec, cosmic rays
$\pi^+ \rightarrow \mu^+\ \nu_\mu$ (mainly)	$\tau = 2.6\ \ 10^{-8}$ sec
K^+	$\tau = 1.2\ \ 10^{-8}$ sec

- particle with short lifetimes :

D^+	$\tau = 1.04\ 10^{-12}$ sec
B^+	$\tau = 1.6\ \ 10^{-12}$ sec
$\Delta^{++} \rightarrow N\ \pi$	$\tau \sim 10^{-23}$ sec

particles which can be directly detected

- The lifetimes are given in the particle rest frame
- What we see is the lifetime in the laboratory rest frame

→ one should take into account the relativistic time dilation

→ In real life one measures lengths in the detector

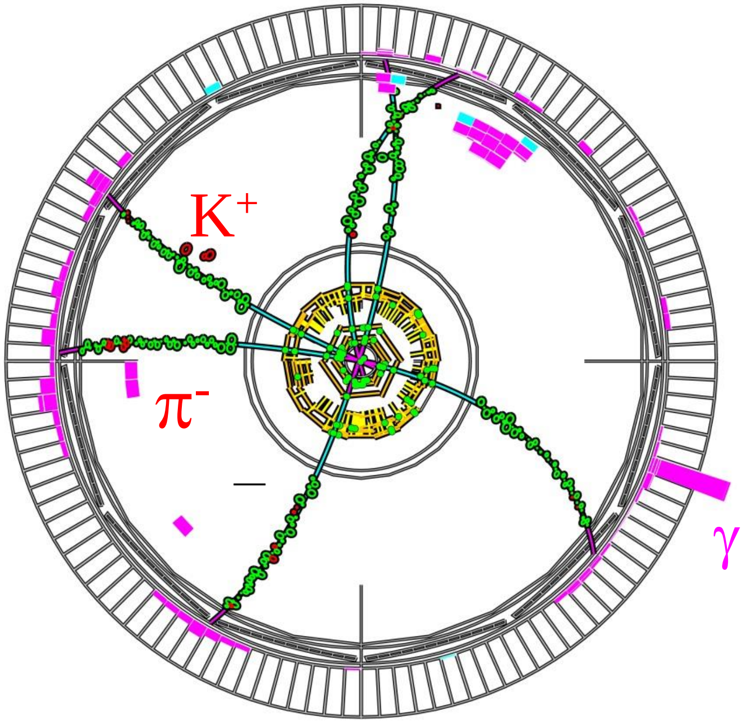
$$\boxed{\begin{array}{llll} L = & \beta\gamma & \times & c\tau \\ & \text{Boost} & \times & \text{lifetime} \end{array}}$$

- Some particles are seen as stable in the detectors.
- Example a pion ($c\tau$ = 7.8m) :

if E_π = 20 GeV → $\gamma = 20/m_\pi = 142.9$;
$\beta = 0.999975$

→ L = 1114.3m

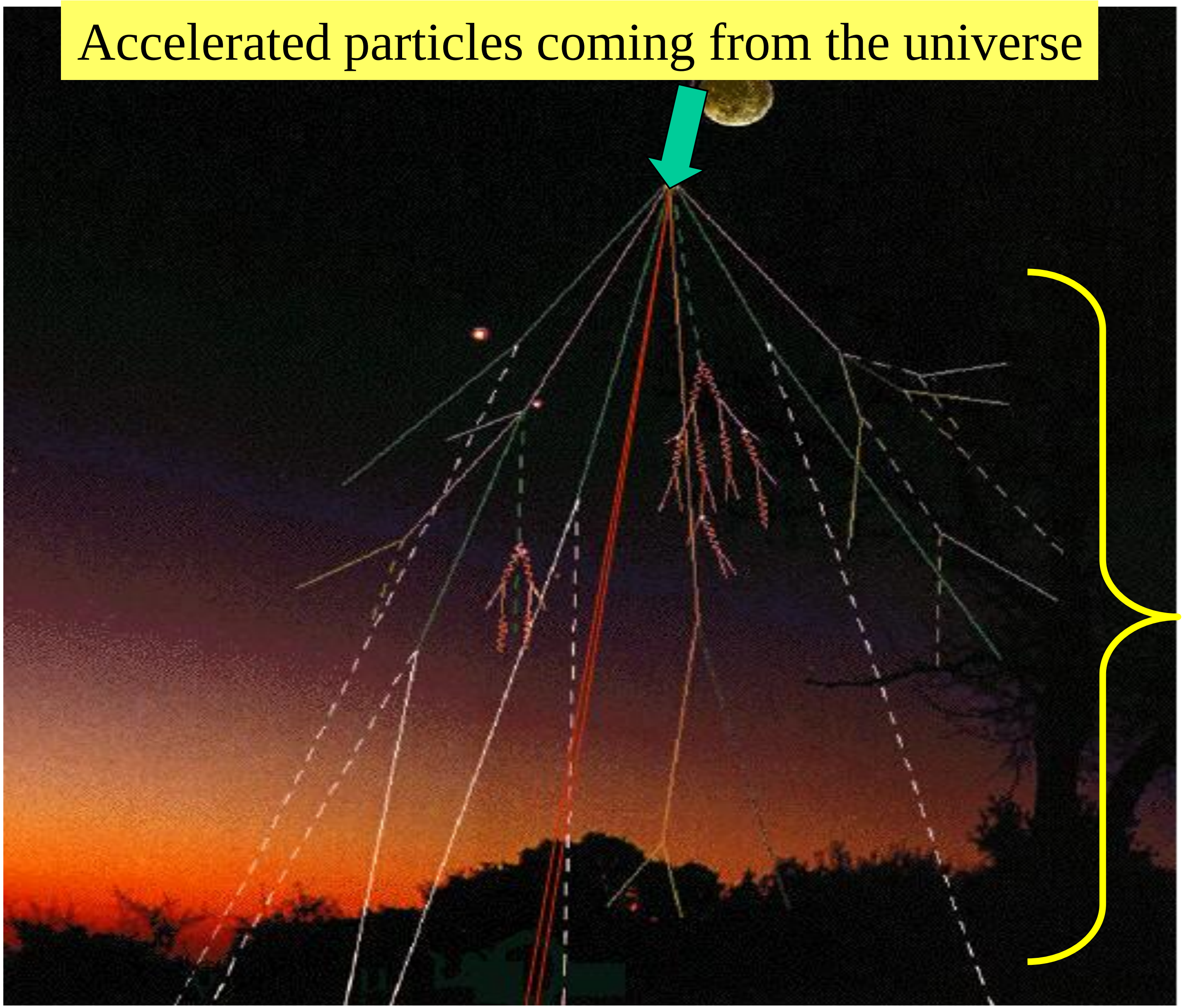
« Event display » of the BELLE experiment
($e^+e^- \rightarrow B\bar{B}$, E_{CM}=10.58 GeV)



$B^0 \longrightarrow K^{*0}\,\gamma$
$\hookrightarrow K^+\,\pi^-$

particles which can be directly detected in the detector : n, γ ,e, p, μ, $\pi^\pm$, $K^\pm$

Cosmic Rays = Cosmic Accelerator

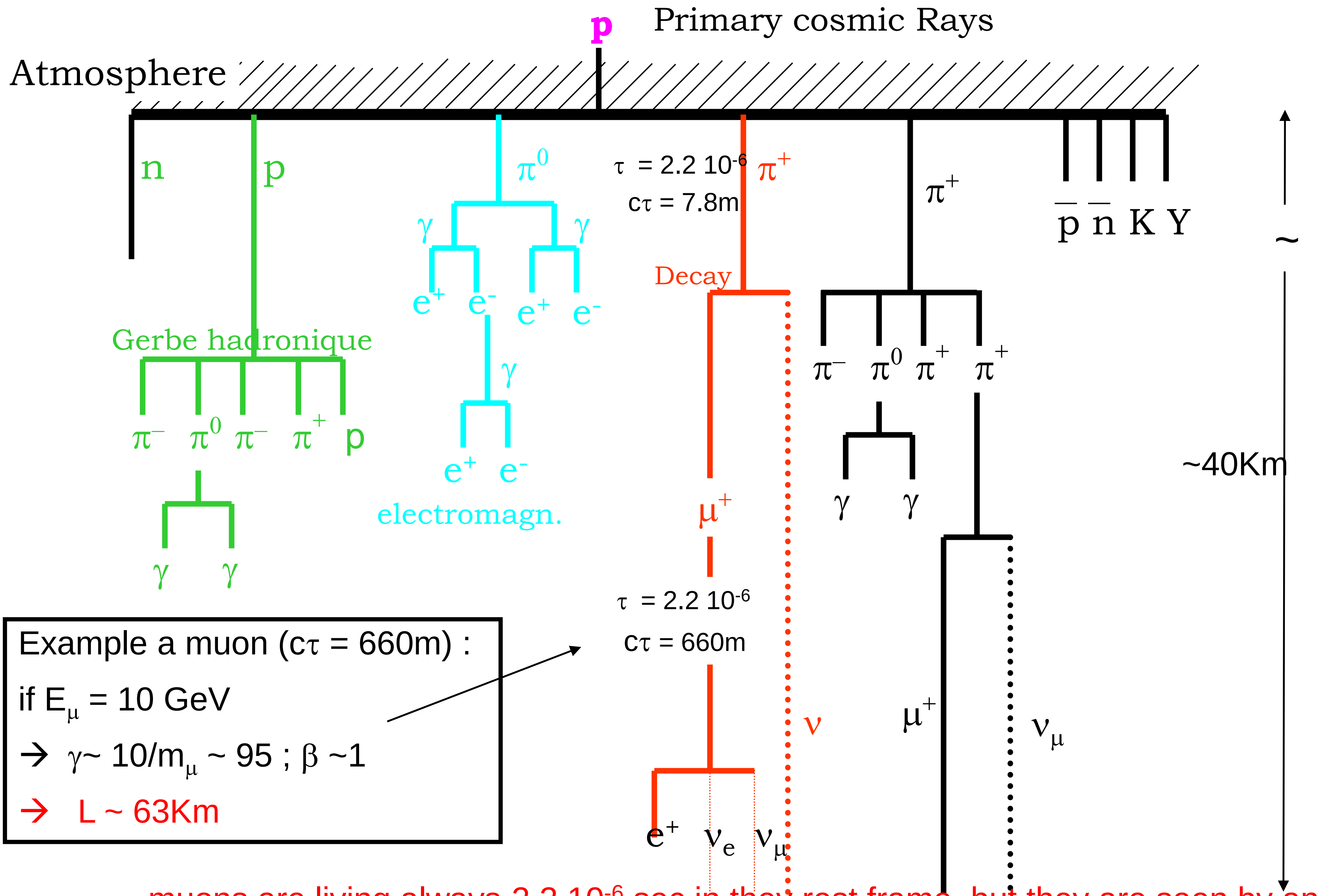


Accelerated particles coming from the universe

Cosmic Shower

↓

Production
of
New Particles



Example a muon ($c\tau$ = 660m) :

if E_μ = 10 GeV

→ γ~ $10/m_\mu$ ~ 95 ; β ~1

→ L ~ 63Km

muons are living always 2.2 10^{-6} sec in they rest frame, but they are seen by an observer as flying a distance much longer than the one ~ to their lifetime

Width : Γ

- The uncertainty principle from Heisenberg for an unstable particle is :

Heisenberg : $\Delta E\ \Delta t \sim \hbar$

$\Delta mc^2 = \Gamma c^2$

τ

Uncertainty on the mass (width Γ) due to τ

By definition : $\Gamma c^2 \equiv \dfrac{\hbar}{\tau}$

The faster the decay, the larger the uncertainty on *m*

Stable particle ↔ well defined mass state

$$\hbar c = 197\ \text{MeV} \times 1\text{fm} \quad ; \quad \hbar = \frac{197 \times 10^{-15}}{3.10^{8}} = 6.582\ 10^{-22}\text{MeV.s}$$

Measuring widths, one is able to have information on very small lifetimes. This is the way one can have information on a phenomenon extremely fast (the fastest in Nature?...) : a particle with a lifetime of 10^{-23} sec)

Decay	**mc²**	**τ**	**Γc^2**
$K^{*0} \rightarrow K^- \pi^+$	892 MeV	$1.3\ 10^{-23}$ s	51 MeV
$\pi^0 \rightarrow \gamma\ \gamma$	135 MeV	$8.4\ 10^{-17}$ s	8 eV
$D_s \rightarrow \phi\pi^+$	1969 MeV	$0.5\ 10^{-12}$ s	10^{-3} eV

Measurable width

Measurable lifetimes

Breit-Wigner

(approximate computations)

- Schrödinger equation (free particle with energy E_0):

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi = E_0\psi$$

$$\Rightarrow \psi = ae^{-\frac{i}{\hbar}E_0 t}$$

$$\Rightarrow \psi = ae^{-i\frac{c^2}{\hbar}m_0 t}$$ (particle rest frame $E_0=m_0c^2$)

  - stable particle : $|\psi(t)|^2 = |\psi(0)|^2 = |a_0|^2$

$\Gamma c^2 \equiv \frac{\hbar}{\tau}$

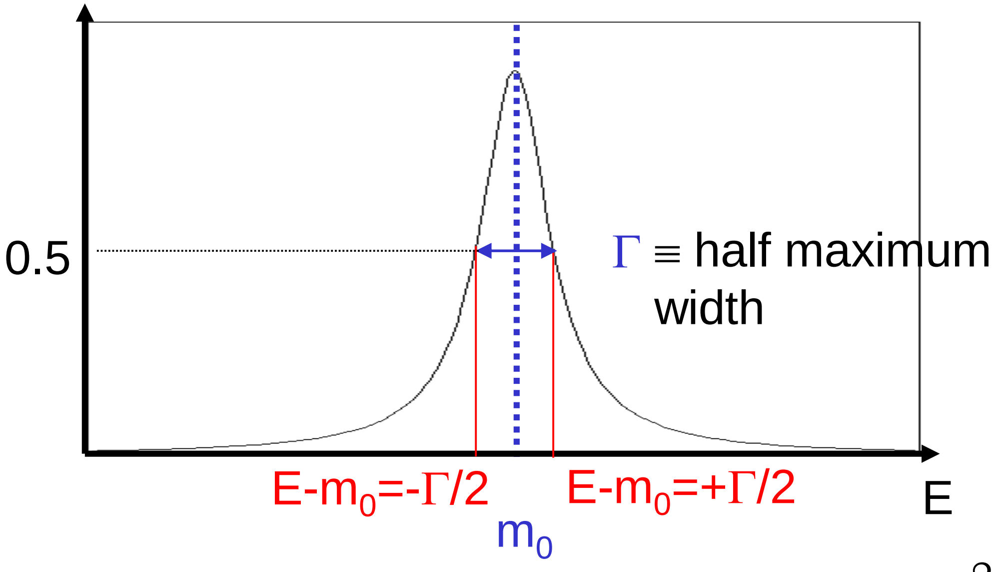
  - unstable particle : $\Rightarrow \psi(t) = a_0 e^{-i\frac{c^2}{\hbar}\left(m_0 - i\frac{\Gamma}{2}\right)t}$ $\Rightarrow a = a_0 e^{-\frac{t}{2\tau}}$ $\Rightarrow |\psi(t)|^2 = |\psi(0)|^2\, e^{-t/\tau}$

We want the probability to find a state of energy E

$$A(E) = \frac{1}{\sqrt{2\pi}}\int_0^{+\infty}\psi(t)e^{\frac{i}{\hbar}Et}dt \propto \frac{1}{\left(E - m_0c^2\right) + i\frac{\Gamma c^2}{2}}$$

Probability = $|A|^2$

$$\Rightarrow |A|^2 \propto \frac{1}{\left(E - m_0c^2\right)^2 + \Gamma^2 c^4/4}$$



Several possible final states (decay modes/channels) :

$\Rightarrow$ branching ratios (BR_i) : probability to obtain a final state i ($\Sigma_i BR_i=1$)
partial width Γ_i (definition) : $BR_i=\Gamma_i/\Gamma$

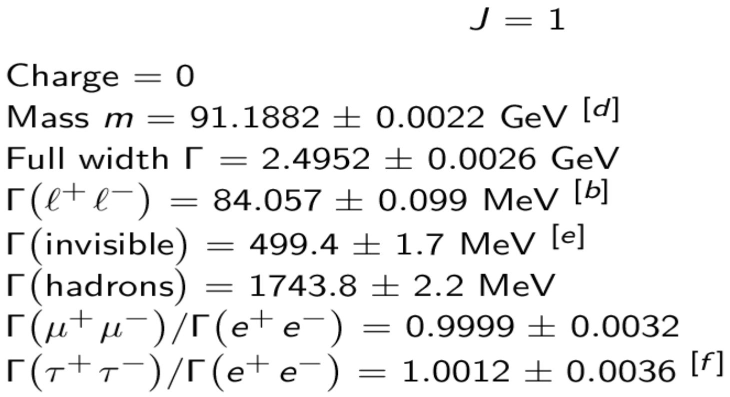
Example:

$\Lambda\rightarrow p\pi$ in 64 % of the cases
$\Lambda\rightarrow n\pi^0$ in 36 % of the cases

Relation between lifetime, partial widths and branching ratios :

$$\boxed{\tau=\frac{\hbar}{c^2}\frac{1}{\Gamma}=\frac{\hbar}{c^2}\frac{BR_i}{\Gamma_i}}$$

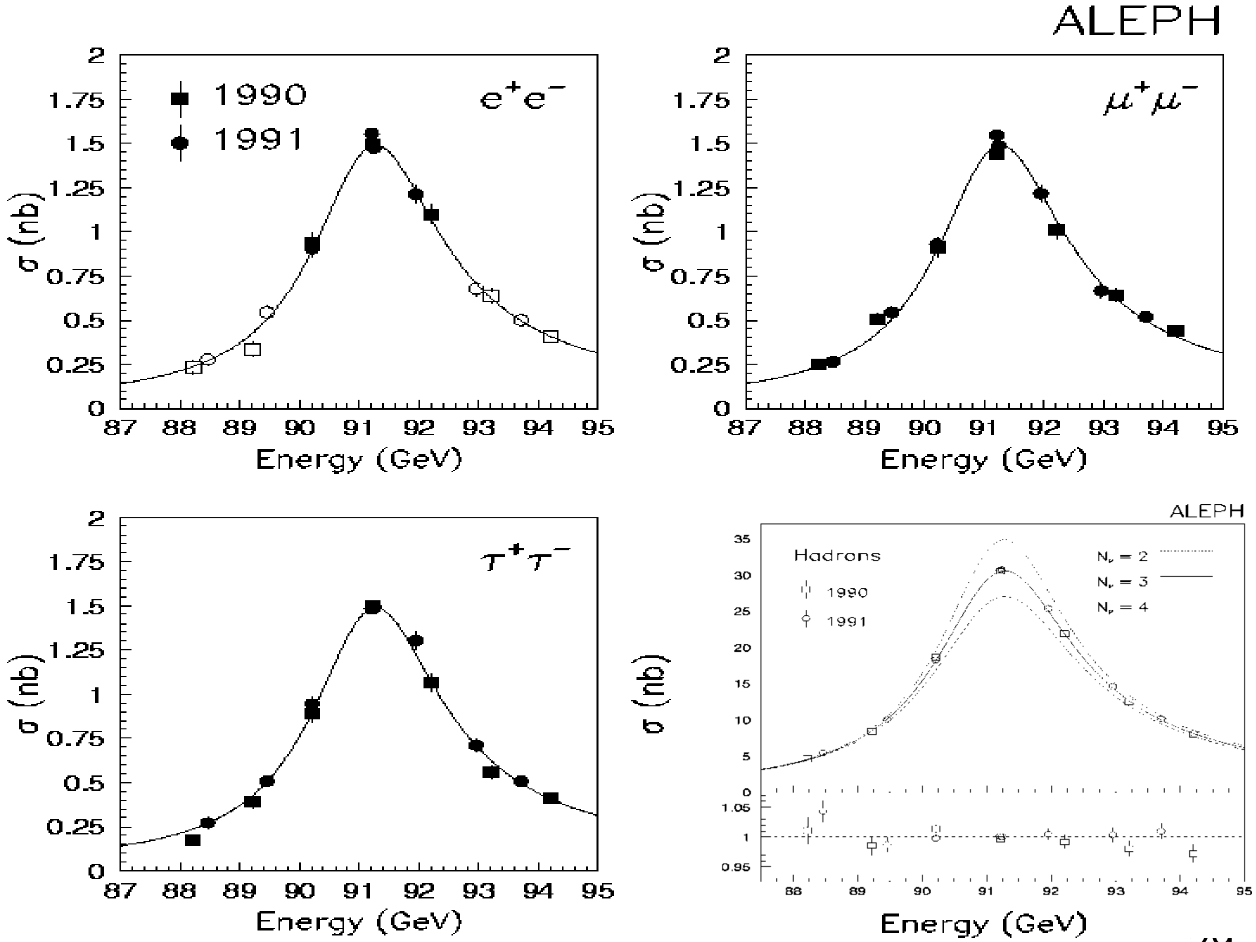
Example : Z^0 partial widths



$J = 1$

Charge = 0
Mass m = 91.1882 ± 0.0022 GeV [d]
Full width Γ = 2.4952 ± 0.0026 GeV
$\Gamma(\ell^+\ell^-)$ = 84.057 ± 0.099 MeV [b]
Γ(invisible) = 499.4 ± 1.7 MeV [e]
Γ(hadrons) = 1743.8 ± 2.2 MeV
$\Gamma(\mu^+\mu^-)/\Gamma(e^+e^-)$ = 0.9999 ± 0.0032
$\Gamma(\tau^+\tau^-)/\Gamma(e^+e^-)$ = 1.0012 ± 0.0036 [f]

You can see that Z^0 in different decay modes has always the same width which is related to his lifetime

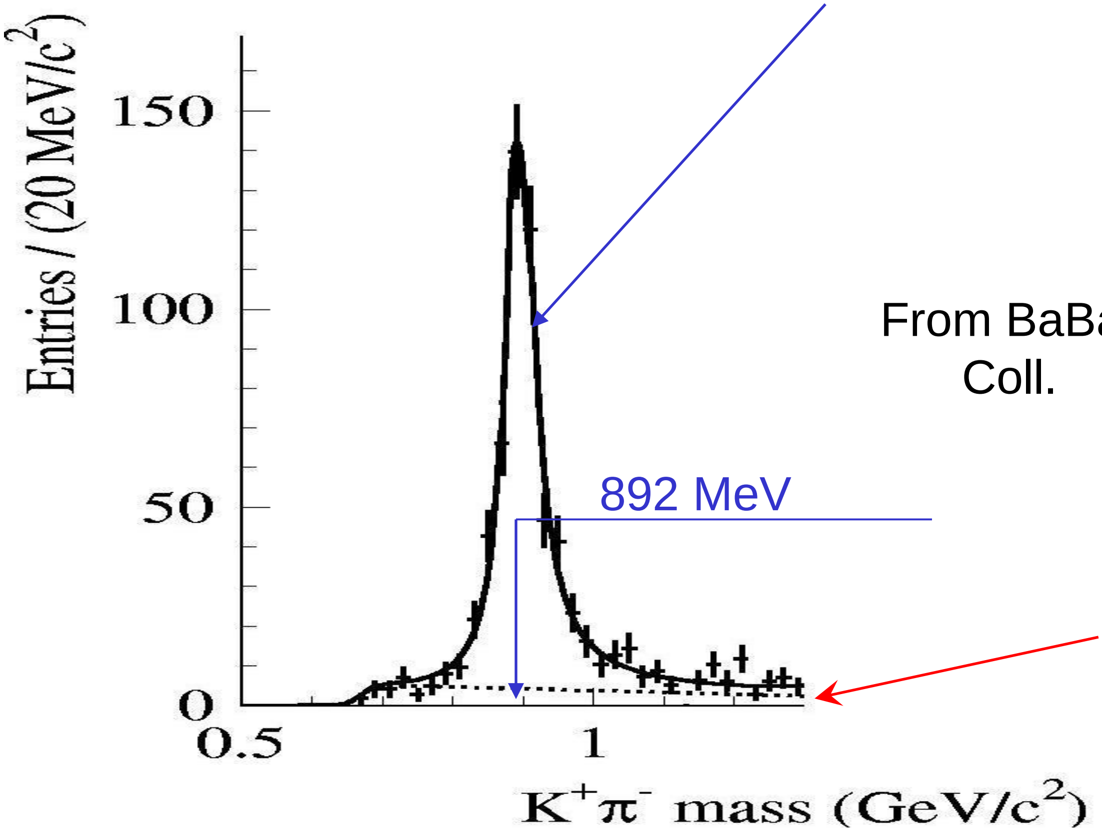


Experimental spectra

experimental spectrum $K^-\pi^+$:

- Search for a K^- and a π^+ in the detector and computation of the invariant mass

Fitted by a Breit-Wigner Γ =51MeV



892 MeV

From BaBar
Coll.

This is a K* !

« combinatorial » background

π^0 experimental spectrum :

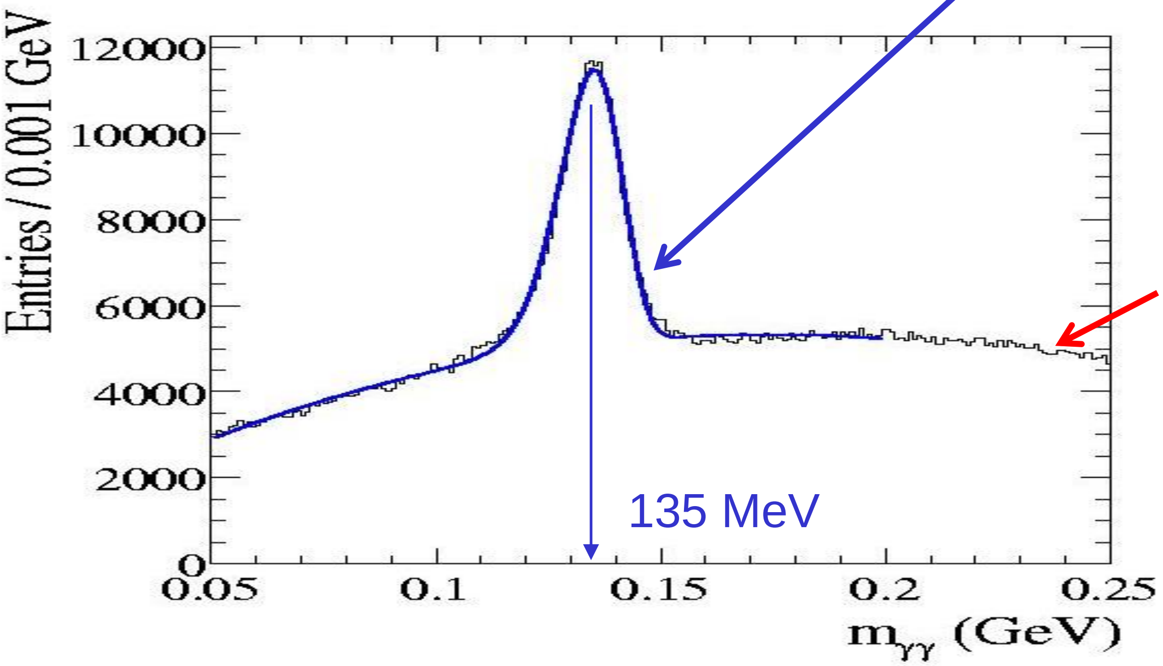
2 γ reconstruction and computation of the invariant mass.

PDG → τ = 8.4 x10^{-17} s ↔ Γ= 8 eV

Fit by a gaussian

σ ~ 7MeV

?



135 MeV

Detector resolution effect

« combinatorial » background

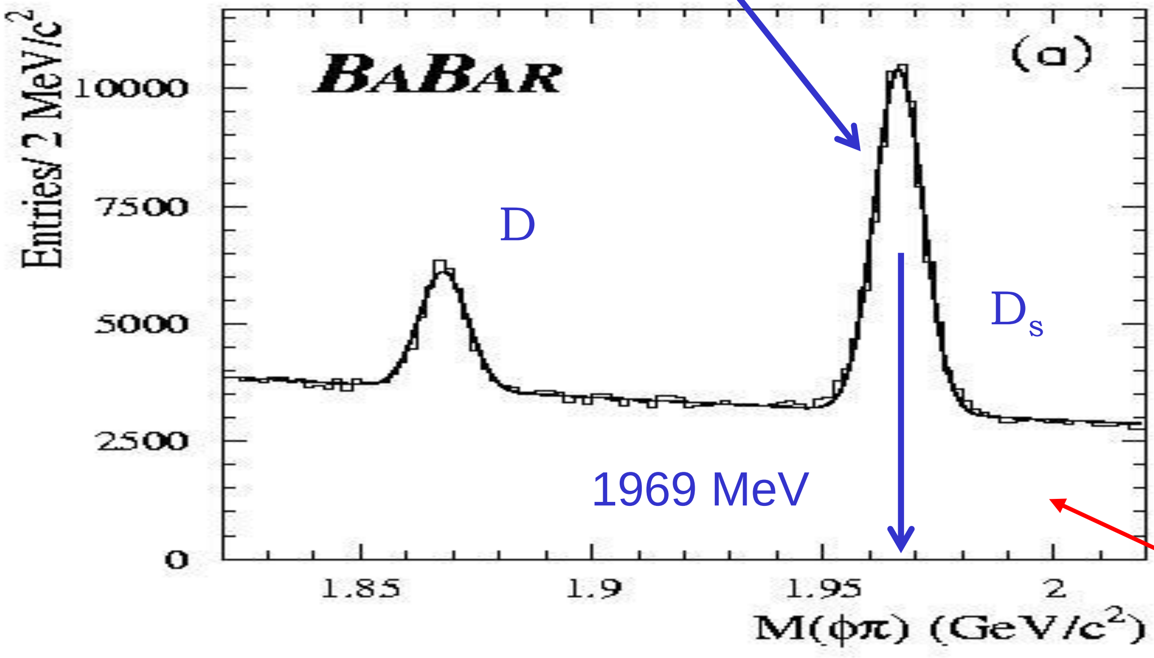
D$_s$ experimental spectrum : (D$_s$ $\rightarrow \phi\pi^+$ and $\phi \rightarrow \pi^+\pi^-$)

PDG → τ = 500 x10^{-15} s ⟷ $\Gamma \sim 10^{-3}$ eV

But one sees >> 10^{-3} eV

Fit by a gaussian $\sigma \sim$ 10MeV

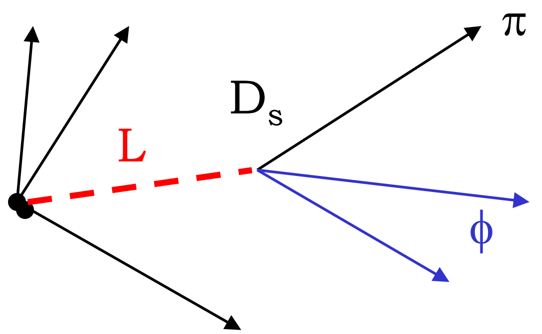
Detector resolution effect



⇒ **One measures directly «long » lifetimes not through widths**

« combinatorial » background

$\tau(D_s)$:

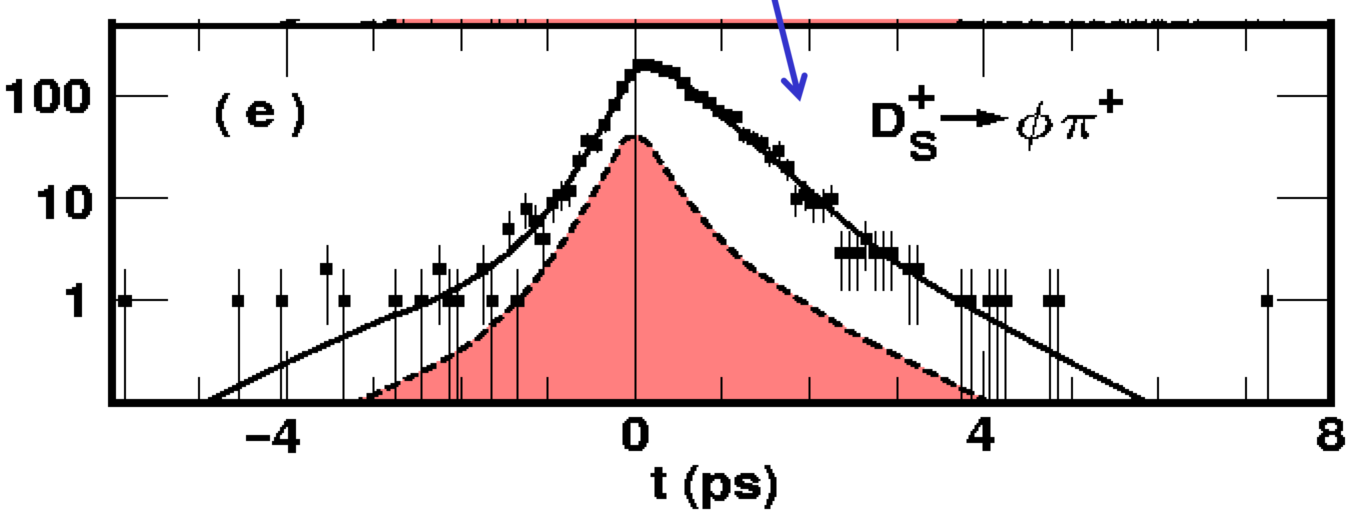


Measurement of the D_s lifetime

$$t = \frac{L \cdot m}{p}$$

t : proper time

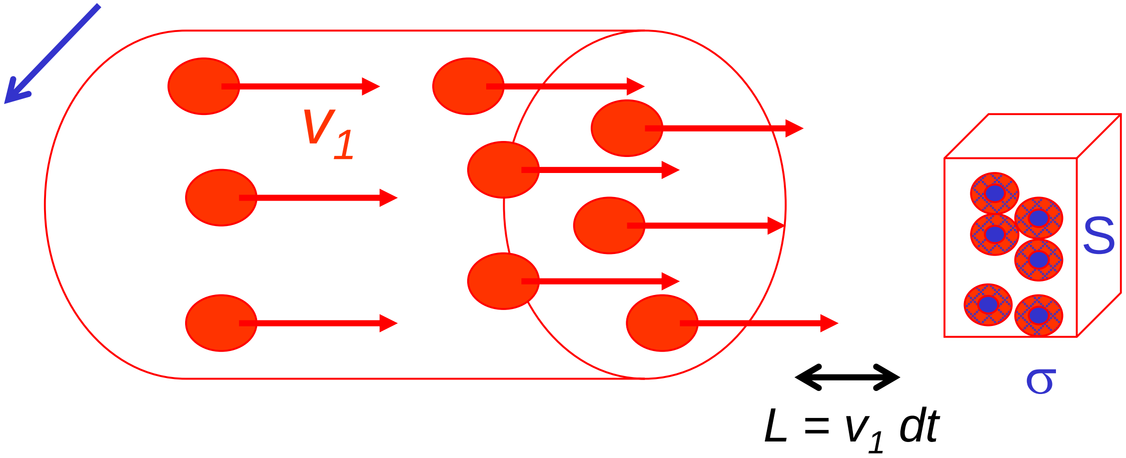
Experiment CLEO : $\tau(D_s) = 486.3 \pm 15.0 \pm 5.0$ fs



Cross Section : σ

Volume of target (*dV*=*Sdx*)

$$\frac{dN_{\text{int}}}{dt} = \quad = F\sigma n_2 dV$$



Flux *F*=number of particle crossing,
in dt, the unity surface ⊥ to *v1*

$F = n_1\ (L/dt) = n_1 v_1$

$$\boxed{\frac{dN_{\text{int}}}{dtdV} = n_1 v_1 n_2 \sigma}$$

The number of interactions per unit of volume and time is thus defined by

- **The physics processes σ** are « hidden » in this term
- The number of particles per unit of volume in the beam (n_1)
- The number of particles per unit of volume in the target (n_2)

- σ : $[L]^2$
- *1 barn* = 10^{-24} cm^2

Parentheis : From cross section → number of produced event : the luminosity

Instantaneous luminosity

$$\boxed{\frac{dN}{dt} = L \cdot \sigma}$$

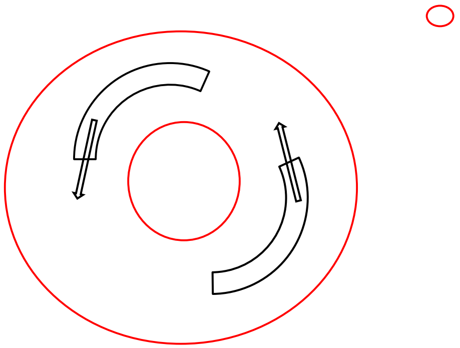
Number of interactions /s

luminosity

$\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$

Cross section

$$\frac{dN_{\mathrm{int}}}{dtdV} = n_1 v_1 n_2 \sigma$$



Colliding mode

$$\frac{dN}{dt\,dV} = \frac{n1}{V}\frac{d}{c}\frac{n2}{V}\ \sigma dV = \frac{n1}{2\pi Rsxsy}\frac{2\pi R}{c} n2\ \sigma = \frac{n1}{sxsy} f\ n2\sigma$$

$$\boxed{L = \frac{kfN_+N_-}{\cdot S_x S_y}}$$

k bunches

f (=c/circumference) frequency

N_+ : number of electrons in a bunch

N_- : number of positrons in a bunch

An example : PEP-2 (where BaBar detector was installed)

$$I(e) = \left[\frac{C}{s}\right]$$

charge

time

$$I(e) = N(e) \times q_e \times N^e_{bunches} \times \frac{c}{L_{circ}}$$

Circumference	2200 m
$I(e^-)$	0.75 A
$I(e^+)$	2.16 A
$N_{paquets}$	2 x 1658
$N(e^-)$/bunch	2.1 10^{10}
$N(e^+)$/bunch	6.0 10^{10}
Beams size	s_x=150 μm, s_y=5 μm

$$L = \frac{kfN_+N_-}{s_x s_y}$$

$\Rightarrow L=3\ 10^{33}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

Macroscopic quantity → relates the microscopic world (σ) to a number of events

$$\boxed{\frac{dN}{dt} = L \cdot \sigma}$$

How to calculate Widths and Cross Sections

More advanced

The total cross section σ for a collision a+b →1+2+…n and the width for a decay Γ a→1+2+… n are given by :

$$d\sigma = \frac{1}{F}\sum_{\text{int}}\left|\langle f|T|i\rangle\right|^2 (2\pi)^4 \delta^4\left(Q_f - Q_i\right)\prod_{k=1}^{n}\frac{d^3 p_k}{(2\pi)^3 2E_k}$$

$$d\Gamma = \frac{1}{2m_a}\sum_{\text{int}}\left|\langle f|T|i\rangle\right|^2 (2\pi)^4 \delta^4\left(Q_f - Q_i\right)\prod_{k=1}^{n}\frac{d^3 p_k}{(2\pi)^3 2E_k}$$

$F :$ flux : $F = 4\sqrt{q_a q_b - m_a^2 m_b^2}$

$\sum_{\text{int}}$: over all the internal degrees of freedom of the final particles

T : depend upon spins and momenta

To be determined from the interaction

$\langle f|T|i\rangle$: matrix element of the transition $|i\rangle \to |f\rangle$

$Q_i = q_a + q_b$ momentum-energy quadrivector energie of the initial state $|i\rangle$

$Q_i = \sum_{k=1}^{n} q_k$ momentum-energy quadrivector energie of the final state $|f\rangle$

$\vec{p}_k, E_k$: momentun and energy of the kth particle in the final state

Differential element :

$$d\phi_n\left(Q_i, q_1, \ldots, q_n\right) = (2\pi)^4 \delta^4\left(Q_f - Q_i\right)\prod_{k=1}^{n}\frac{d^3 p_k}{(2\pi)^3 2E_k}$$

Named phase space at n-body

This term contains all the kinematics

$$d^3 p = p^2 dp d\Omega$$

$$d\Omega = d\cos\theta d\phi$$

Taking the case of :a+b →1+2 ou a→1+2 we write :

$$\sigma = \int_{impulsion} \underbrace{\int_{spin}}_{\frac{1}{2s_a+1}\cdot\frac{1}{2s_b+1}} \sigma$$

Not polarised beam

We obtain

$$\begin{cases} \left.\dfrac{d\sigma}{d\Omega^*}\right|_{a+b\to1,2} = \boxed{\dfrac{1}{64\pi^2}\cdot\dfrac{1}{s}\cdot\dfrac{p_f^*}{p_i^*}} \sum_{\text{int}}\left|T_{fi}\right|^2 \\ \left.\dfrac{d\Gamma}{d\Omega^*}\right|_{a\to1,2} = \boxed{\dfrac{1}{32\pi^2}\cdot\dfrac{p_f^*}{m_a^2}} \sum_{\text{int}}\left|T_{fi}\right|^2 \end{cases}$$

From « Feynman » diagram..

Kinematics

$$a+b\to1+2$$

$$E_1^* = \frac{m_1^2-m_2^2+s}{2\sqrt{s}} \quad ; \quad E_2^* = \frac{m_2^2-m_1^2+s}{2\sqrt{s}} \quad \Rightarrow p^*$$

$$a\to1+2$$

In a decay we have $s=m_a^2$

If we suppose that $m_1=m_2=m$

$$p^* = \sqrt{\left(\frac{m_a}{2}\right)^2 - m^2}$$

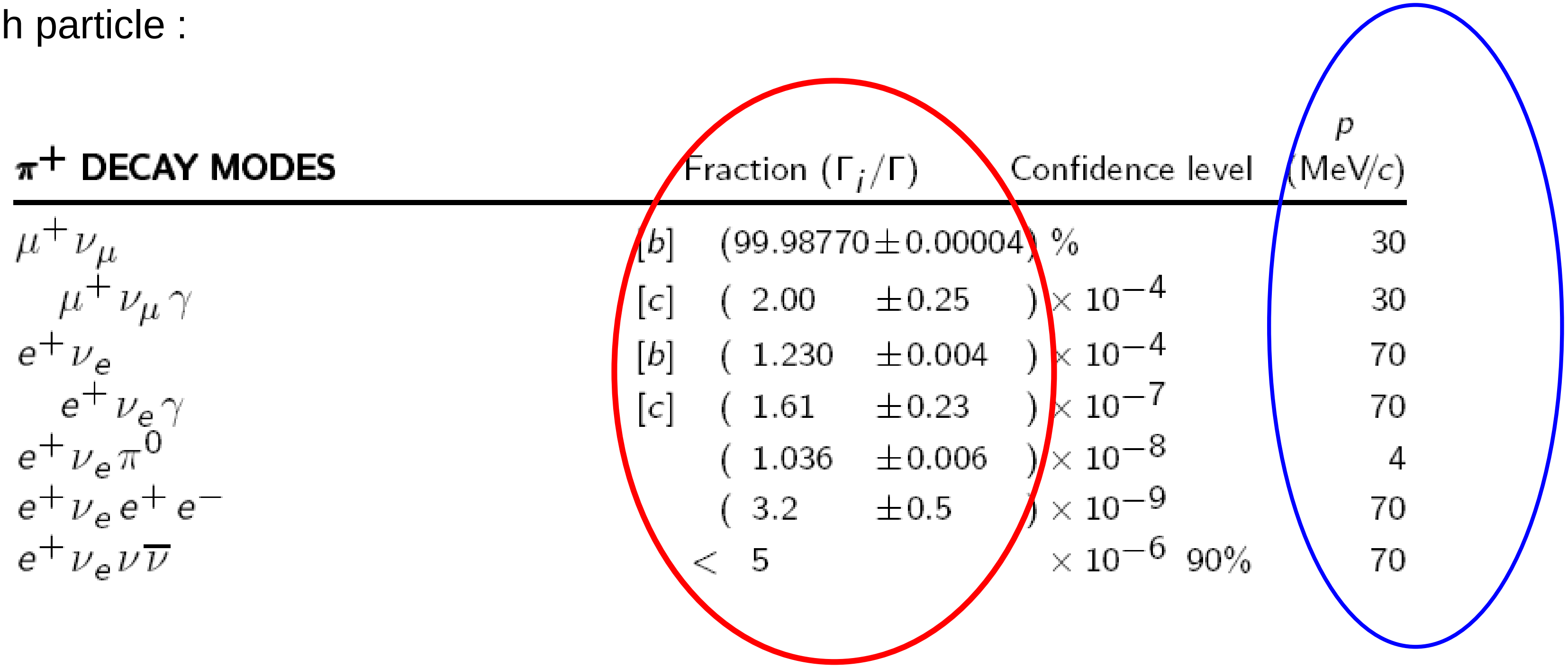
if we neglect the masses of the final particles $m_{1,2} \ll m_a$

$$p^* = \frac{m_a}{2}$$

The **P**article **D**ata **G**roup book : where all the measured particles properties are recorded

(paper or internet)

For each particle :



π^+ DECAY MODES		Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\mu^+\nu_\mu$	[b]	(99.98770 ± 0.00004) %		30
$\mu^+\nu_\mu\gamma$	[c]	$(2.00\pm0.25)\times10^{-4}$		30
$e^+\nu_e$	[b]	$(1.230\pm0.004)\times10^{-4}$		70
$e^+\nu_e\gamma$	[c]	$(1.61\pm0.23)\times10^{-7}$		70
$e^+\nu_e\pi^0$		$(1.036\pm0.006)\times10^{-8}$		4
$e^+\nu_e e^+e^-$		$(3.2\pm0.5)\times10^{-9}$		70
$e^+\nu_e\nu\overline{\nu}$		$<5\times10^{-6}$	90%	70

This is $BR_i=\Gamma_i/\Gamma$

$$\boxed{\tau=\frac{\hbar}{c^2}\frac{1}{\Gamma}=\frac{\hbar}{c^2}\frac{BR_i}{\Gamma_i}}$$

This is the p_{max} from the previous page. Obtained by 4-momentum conservation

$$E_\mu^{\;*}=\frac{m_\pi^2+m_\mu^2-m_\nu^2}{2m_\pi}$$

$$p_\mu^{\;*}=\sqrt{E_\mu^{*2}-m_\mu^{*2}}$$

p_{max} ~30 MeV

Short example ... Can be much longer !

I.4

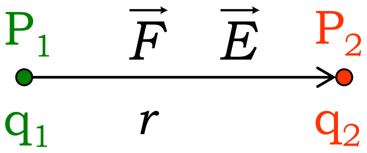
Introduction to the interactions

Interactions : introduction

Classical physics :

The particle P_1 creates around it a force field. If one introduces the particle P_2 in this field it undergoes the force.

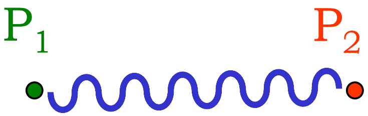
Electrostatic example :



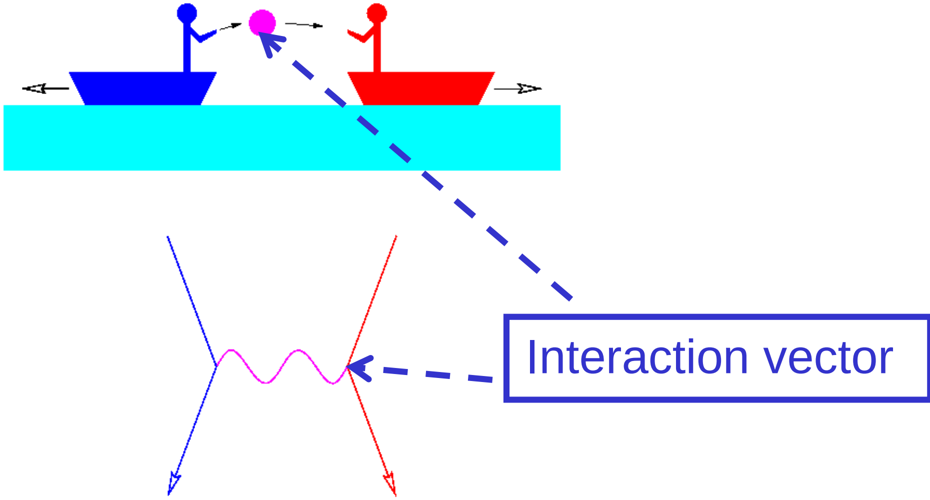
$$\vec{F} = q_2\vec{E}(r) = q_2\frac{kq_1}{r^2}\vec{u}_r$$

«modern» physics:

P_1 and P_2 exchange a field quantum; the interaction boson



The **heavier** the ball, the more difficult it will be to throw it **far away**



Range of the interaction $\propto$1/mass of the vector

Range of an interaction

Not exact. Heuristic

- Creation and exchange of an interaction particle
  ⇒ violation of the energy conservation principle during a limited time

$$\Delta t \approx \frac{\hbar}{\Delta E} = \frac{\hbar}{mc^2}$$

Heisenberg principle

- During Δt the particle can travel $R = c\,\Delta t$

$$R = \frac{\hbar c}{mc^2}$$

Range → « reduced » wave length (Compton)

with $\hbar c \cong 197.3$ *MeV fm*

Example : an interaction particle with $m = 200$ *MeV* ⇔ $R = 1$ *fm*

Shape of the interaction potential

Klein-Gordon equation for a spin 0 particle :

$$E^2 = p^2c^2 + m^2c^4$$

operators

$$E = i\hbar\frac{\partial}{\partial t}$$

$$p = -i\hbar\nabla$$

$$(i\hbar)^2\frac{\partial^2\psi}{\partial t^2} = (i\hbar)^2 c^2\nabla^2\psi + m^2c^4\psi$$

$$-\frac{\partial^2\psi}{\partial t^2} = -c^2\nabla^2\psi + \frac{m^2c^4}{\hbar^2}\psi$$

$$\Rightarrow \nabla^2\psi - \frac{m^2c^2}{\hbar^2}\psi - \cancel{\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}} = 0$$

(one only deals with stationary states)

$$\boxed{\nabla^2\psi - \frac{m^2c^2}{\hbar^2}\psi = 0}$$

In spherical symmetry : ψ=U(r) and ΔU(r)=∇^2U(r)= $\boxed{\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU(r)}{dr}\right) = \frac{m^2c^2}{\hbar^2}U(r)}$

if $m \neq 0$:

$$U(r) = -\frac{g^2}{r}e^{-r/R}$$

$r>0$

Yukawa potential

g coupling constant

$$R = \frac{\hbar}{mc}$$

Range

if $m=0$:

$$\Delta U(r) = 0$$

$$U(r) = -\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$r>0$

q_i = charge

In this case the Yukawa potential is equivalent to the Coulomb one

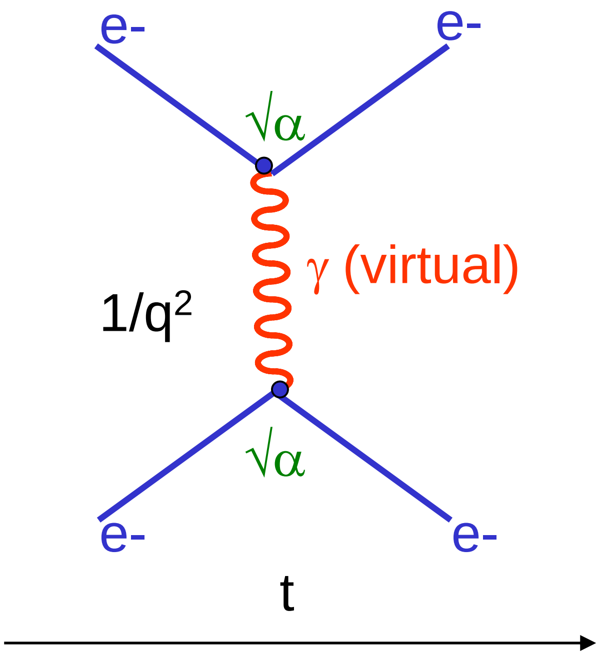
Force	**Relative intensity (order of magnitude)**	**Vector**	**Lifetime (order of magnitude)**
Strong	1	Gluons	10^{-24} s
electromagnetic	10^{-2}	Photon	10^{-19} - 10^{-20} s
Weak	10^{-5}	W and Z^0	10^{-16} - 10^{+3} s
Gravitation	10^{-40}	Graviton	???

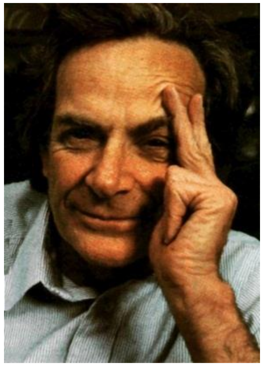
For the strong, electromagnetic and gravitational interactions these orders of magnitudes can be obtained comparing the binding energy of 2 protons separated by ~1fm

The intensity of the interactions dictates the particles lifetimes and their interaction cross sections.

Electromagnetism (QED)

- Between charged particles
- Vector of the interaction : the photon (γ)
- One Feynman graph for QED:





R. Feynman

electrons exchanging a photon

or

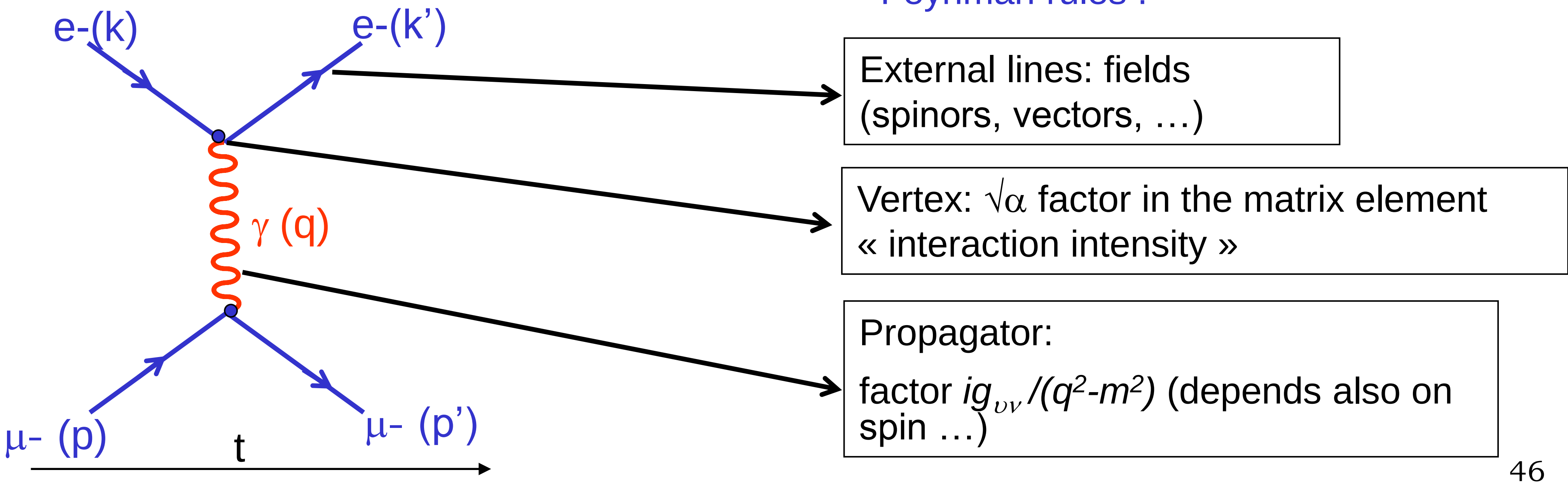
An e^- which emits a γ and moves back. The γ is absorbed by an other e^- whose direction is modified

Feynman graph

- A powerful « graphical » method to display the interaction in perturbations theory (each diagram is a term in the perturbation series)
- Each graph is equivalent to « a number »
- → computation of the matrix elements and of the transition probabilities

- Horizontal axis : the time
- Lines are particles which propagate in space-time
- The ● represent the vertices «location» of the interaction (where there is quantum number conservation)

Feynman rules :



External lines: fields (spinors, vectors, …)

Vertex: √α factor in the matrix element « interaction intensity »

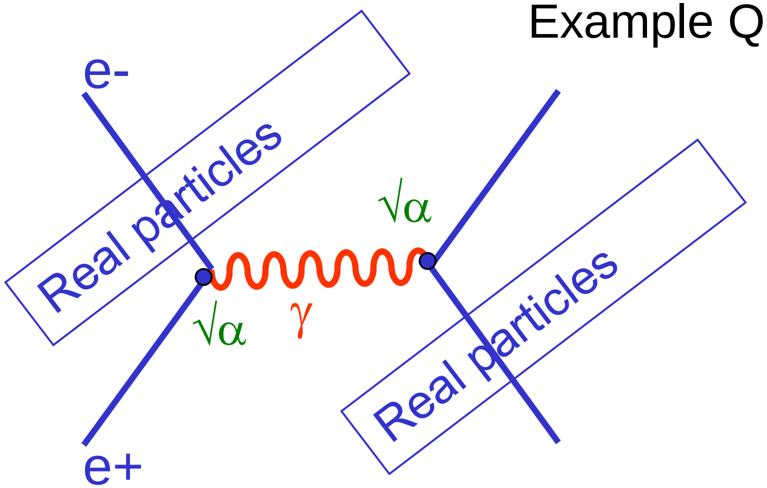
Propagator:

factor $ig_{\upsilon\nu}/(q^2-m^2)$ (depends also on spin …)

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137}$$

Virtual particles

Example QED : e^+e^- symmetric collision in the rest frame



$$E_{e+} + E_{e-} = E_\gamma$$

$$\overrightarrow{p_+} + \overrightarrow{p_-} = \overrightarrow{p_\gamma}$$

$$m_\gamma^2 = 2m_e^2 + 2E_{e+}E_{e-} - 2p_+p_-\cos\theta$$

In the rest frame : $\overrightarrow{p_+} + \overrightarrow{p_-} = \overrightarrow{p_\gamma} = \vec{0}$

$$\theta = \pi \Rightarrow m_\gamma^2 = 2m_e^2 + 2E_{e+}E_{e-} + 2p_+p_-$$

incompatible with $m_\gamma = 0$

The γ is « off-shell »

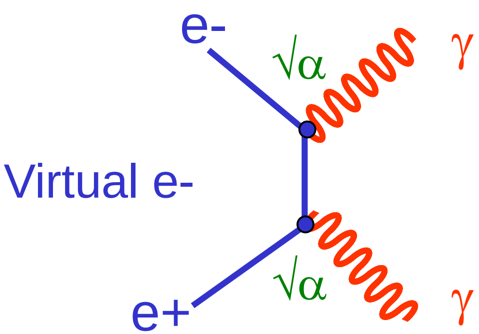
It can be interpreted as :

Violation of the energy-momentum conservation law

Or

Creation of a massive virtual photon during a « short » time

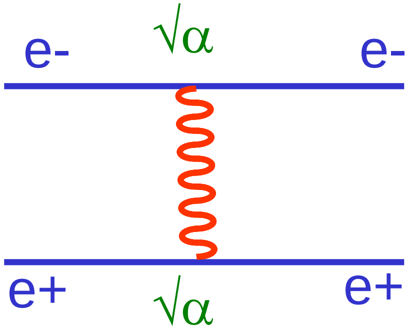
the γ can only exist virtually thanks to $\Delta E.\Delta t \approx \hbar$



2 γ production going in opposite directions

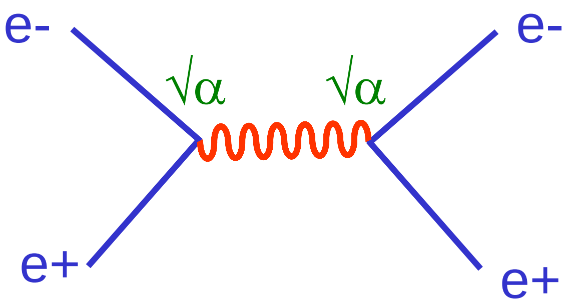
→ energy-momentum conservation

$e^+e^- \rightarrow e^+e^-$ interaction



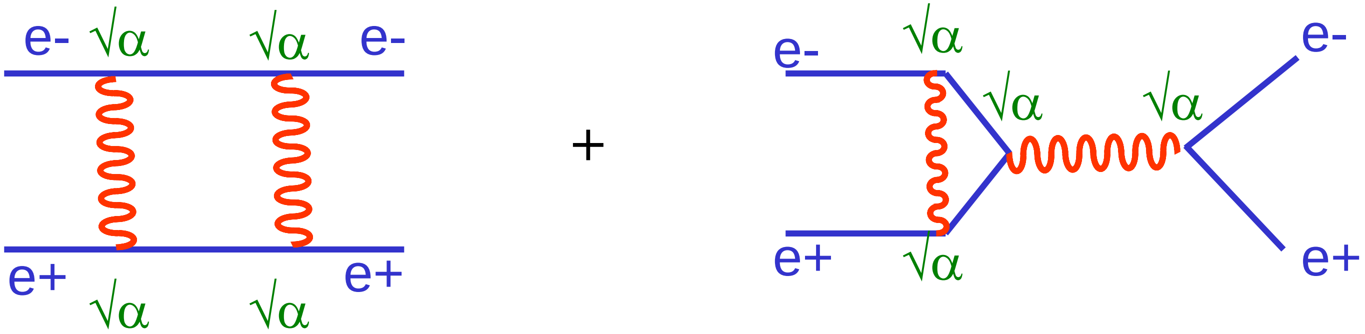
γ exchange between an e+ and an e-

+



e+e- pair annihilation in γ and γ conversion in an e+ and an e-

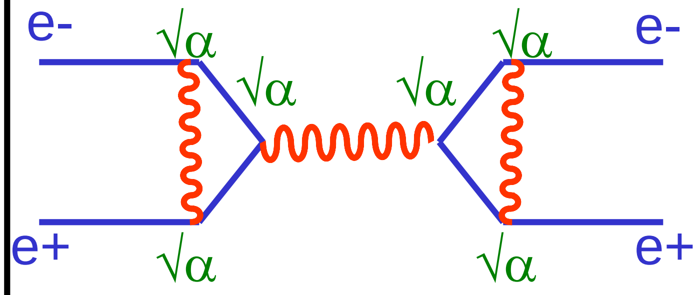
$\sim\alpha$



+ …

$\sim\alpha^2$

exchange of 2 γ between an e+ and an e-



+ …

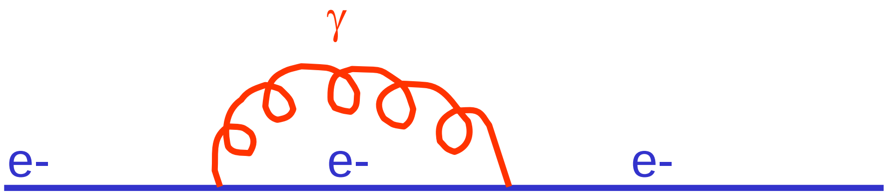
$\sim\alpha^3$

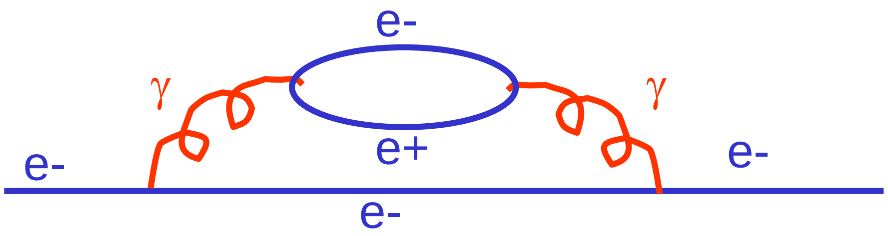
α small (1/137) : one can develop in perturbation series

The way we see the electron and the photon is modified

electron :

The electron emits and absorbs all the time virtual γ, it can be seen as :

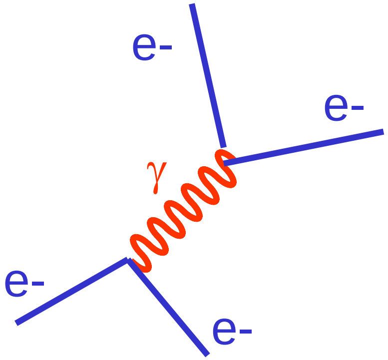


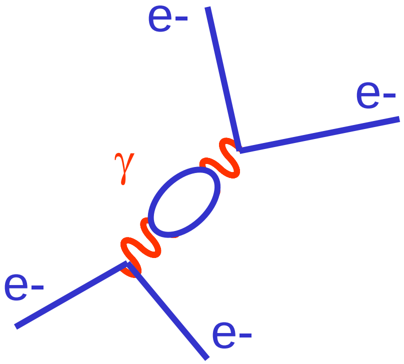


…

=> Theoretical (α « running »), Vacuum polarization and experimental (g-2) consequences

photon :

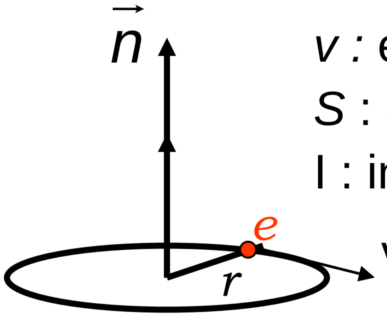




(g-2) : Experimental evidence of the vacuum polarisation

Gyro-magnetic ratio g

- The magnetic moment associated associated to the angular momentum of the electron



n : unity vector
v : electron speed
S : surface
I : intensity = charge / time

$$\vec{\mu} = I\ S\ \vec{n} = \frac{e}{\frac{2\pi r}{v}} \pi r^2 \vec{n} = \frac{e}{2m}(mvr)\vec{n}$$

Angular momentum $\hbar\ell$

$$\mu = \mu_B \ell \quad \text{with} \quad \boxed{\mu_B = \frac{e\hbar}{2m}}$$

Bohr magneton

$$\vec{\mu} = \mu_B \vec{L}$$

- Intrinsic magnetic momentum :

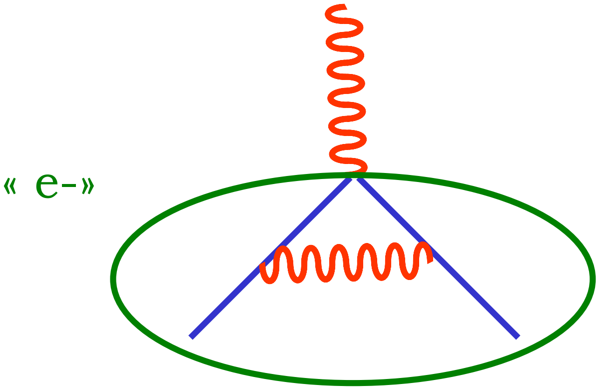
$$\vec{\mu} = g\,\mu_B \vec{S}$$

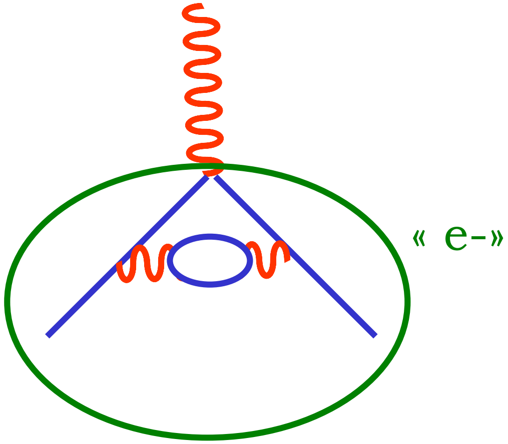
spin

gyro-magnetic spin ratio

Dirac : for spin ½ point-like particles : *g*=2

The value of *g* is modified by :





+ …

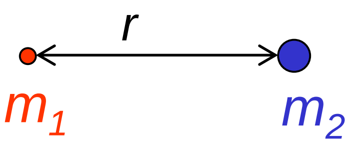
One defines $a = \frac{g-2}{2} = \frac{g}{2} - 1 = \frac{\alpha}{2\pi} + \ldots \approx \frac{1}{800}$

a=0.00115965241 ± 0.00000000020 experiment (10^{-11} precision)

a=0.00115965238 ±0.00000000026 theory (α^3)

Gravitational Force

$$F = \frac{Gm_1m_2}{r^2}$$



$$G = 6.67259(85)10^{-11}\frac{\mathrm{m}^3}{\mathrm{kg\ sec}^2}$$

Newton constant

To compare with the electromagnetic force for the hydrogen atom

$$\frac{e^2}{4\pi\varepsilon_0\hbar c} = \alpha \approx \frac{1}{137}$$

$$\frac{Gm_em_p}{\hbar c} = \alpha_{grav} \approx 3.3\times10^{-42}$$

The effects of gravitation are very small at the atom scale ➔ neglected..

- Important effects if α_{grav} ~1

$$\frac{Gm^2}{\hbar c} \sim 1 \Rightarrow mc^2 \sim 10^{19}GeV$$

Masse de Planck

- For energies much lower than 10^{19} GeV we can neglect gravitational effects
- Actually there is not satisfactory theory for gravitation

Interactions : summary

- The interactions are mediated by vector bosons
  interaction range $\propto$ 1/mass
- Feynman graph = display of a matrix element of the transition in the perturbations series framework
- Virtual particles (off-shell particles during a short time)

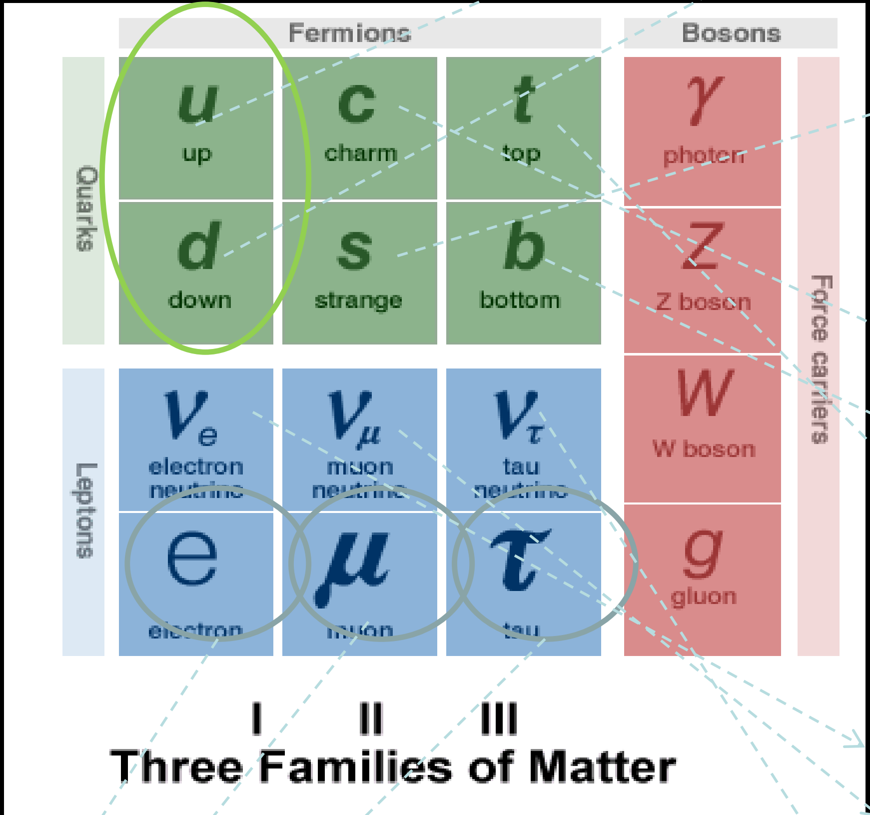
- QED: electric charge, γ, vacuum polarisation, α ↗ with energy

Strong interaction (discussed in a devoted lectures)

Weak interaction (discussed in devoted lectures)

- QCD: colour, gluons (self-interaction), α_s ↘ with energy (asymptotic freedom)
- Weak: concerns all fermions, $W^{\pm}$,Z^0

Complements



1912 (*Rutherford*) proton

1932 (*Chadwick*) neutron

1947 Evidence of Strange particles

1970 (SLAC) Quarks discovered on deep inelastic scattering

1974 (Richter/Ting) J/ψ (cc) : quark c

1977 (Ledermann) (bb) : quark b

1994 (CDF/DØ coll.) quark t

1958 (*Reines-Cowan)* (th. Introduced in 1930) ν_e

1964 ν_μ

2000 (*Donut coll.)* ν_μ

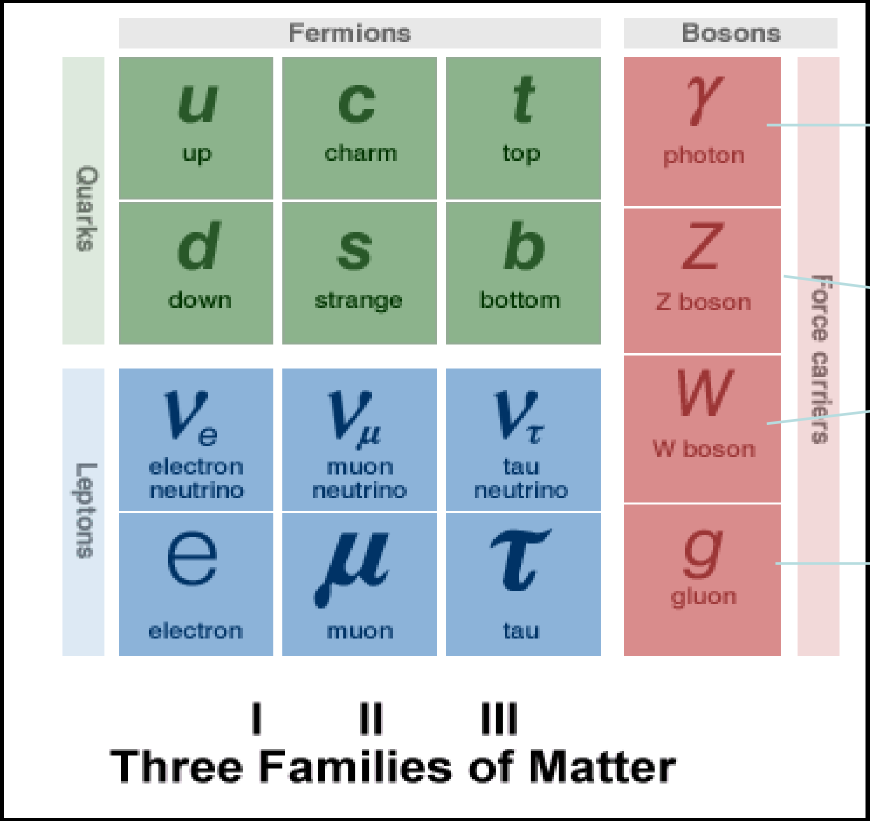
1897 (*Thompson)*

1937 (*Andersson)*

1975 (*Perl/SLAC)*

+ Antiparticles : **1932** Evidence of positron

Interaction is transported by particles



	Fermions			Bosons	
Quarks	*u* up	*c* charm	*t* top	γ photon	Force carriers
	d down	*s* strange	*b* bottom	*Z* Z boson	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	*W* W boson	
	e electron	μ muon	τ tau	*g* gluon	
	I	II	III		

Three Families of Matter

1905 photoelectric effect

1983 (CERN@SPS)

1975 (DESY/SLAC)
Three jets observation

+ **H**
BEH « Higgs »

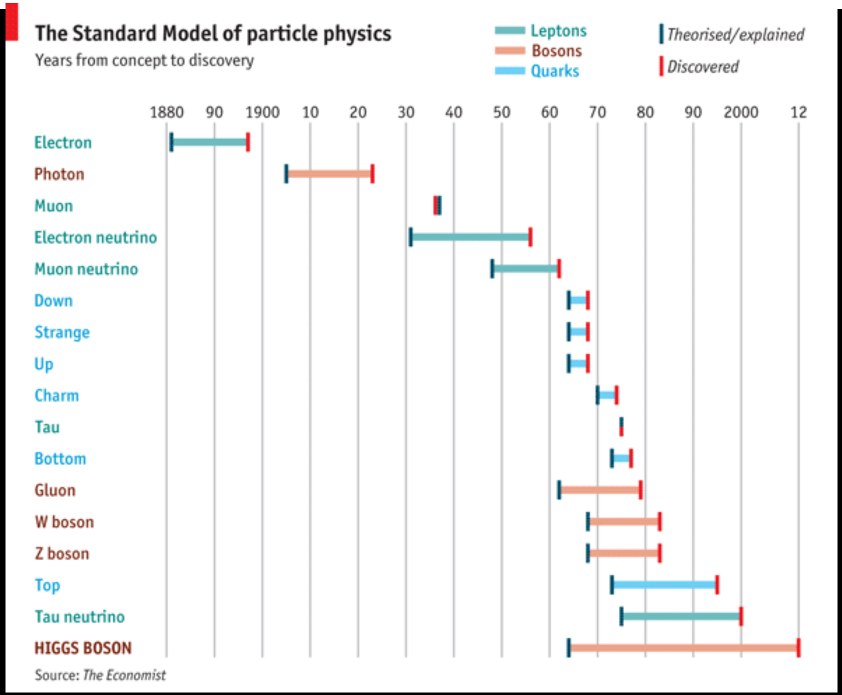
2012
CERN ATLAS/CMS)

> 100 years of search and discoveries

All our knowledge is today « codified » in the **Standard Model** :
Matter, Interaction, Unification Interaction, Unification

•The Standard Model :

- Classify the matter particles in family (fermions)
- Explain the interactions through local gauge principle symmetry (bosons)
- Allow the particle to acquire masses through the **Higgs mechanism**



The Standard Model of particle physics

Years from concept to discovery

Source: *The Economist*

History of the particle predictions and discovery

As you can see for some a lot of time passed from the prediction to the discovery. Often conceptual and technological breakthroughs were needed

For Higgs ...almost 50 years !

APPENDIX I : Angular momentum and spin

Angular momentum

Classical mechanics :

$$\vec{L} = \vec{r} \wedge \vec{p}$$

3 components :

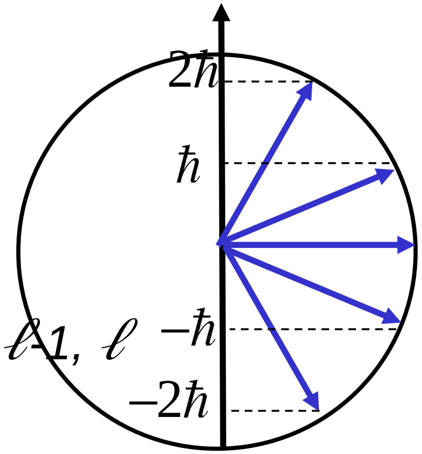
- can be measured with infinite precision
- can have all values

Quantum Mechanics :

same definition, with the operators R and P (notation ***L*** or $\vec{L}$)

- The algebra of the components of **L** : $[L_i,L_j] = i\,\varepsilon_{ijk}L_k$; ε_{ijk}= 0, +1, -1 according to *ijk.* One also has : $L^2=L_i^2+L_j^2+L_k^2$; $[L^2,L_i]=0$
- 2 independent operators (usually : L^2 et L_z)

  (2 useful quantum numbers)



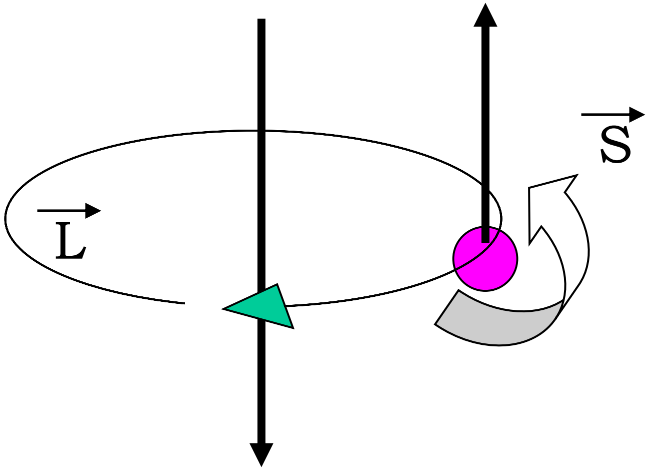
- quantification :
  - L^2 : $\ell(\ell+1)\,\hbar^2$; ℓ is an integer
  - L_z: $m\hbar$ with $m = -\ell, -\ell+1, \ldots, -1, 0, 1, \ldots, \ell-1, \ell$

- Addition of 2 angular momenta :

$$\boxed{|j_1,j_2;m_1,m_2\rangle = \sum_{J=|j_1-j_2|}^{j_1+j_2}\sum_{M=-J}^{J}|J,M\rangle\langle J,M|j_1,j_2;m_1,m_2\rangle}$$

Clebsch-Gordan(CG) coefficients

Spin



- The spin is the intrinsic kinetic momentum of a particle.
- it can be half-integer
- It determines the behavior of a given particle.
- Few examples of experimental evidences for the spin :
  - **Fine structure of the atoms spectral lines** : each line is made of several components very close in frequency
  - **"Abnormal » Zeeman effect** : Each spectral line is divided in a given number of equidistant lines when the atom is in an uniform magnetic field. «Anomaly» : the atoms of Z odd (ex. Hydrogen) divide into an even number of sub-level. In fact the number of levels is $2\ell+1$ → proof of half integer kinetic momentum !

- The spin has no classical equivalent. Trying to explain it saying that the particle rotates on its own axis does not work.

e,p,n have very different characteristics (charge/ mass/interaction) but they have the same spin : ½

- The spin obeys the same laws as the other kinetic momenta :
  - Algebra similar as the **L** one
  - S^2 can have the values $s(s+1)\hbar^2$ (s can be half integer)
  - And S_z : $m\hbar$ with $m=-s,-s+1,\ldots-1,0,1,\ldots,s-1,s$
  - One can add a spin with
    - An other spin ($S = S_1 \oplus S_2$)
    - With an total angular momentum ($J = L \oplus S$)

A particle can have any angular momentum *L* but its spin *S* is fixed

	integer spin (Bosons)		**Half integer spin (Fermions)**	
	spin 0	spin 1	spin 1/2	spin 3/2
Elementary	-	Vectors of the interactions	quarks, leptons	-
Composite	pseudo-scalar mesons (p,K..)	Vector mesons (ρ,K^*)	some baryons (octet)	some baryons (decuplet)

spin/statistics theorem (Pauli 1940)

Pauli's exclusion principle *: two particles of half integer spin (fermions) cannot be simultaneously in the same quantum state*

Bohr and Pauli

Pauli's principle ↔ anti-symmetry of the wave function by the exchange of 2 particles (for the fermions)

For 2 particles one in the state ψ_α, the other one in the state ψ_β, one can write :

$$\psi(1,2) = \frac{1}{\sqrt{2}}\left(\psi_\alpha(1)\psi_\beta(2) + \psi_\beta(1)\psi_\alpha(2)\right)$$

Symmetric (bosons)

$$\psi(1,2) = \frac{1}{\sqrt{2}}\left(\psi_\alpha(1)\psi_\beta(2) - \psi_\beta(1)\psi_\alpha(2)\right)$$

anti-symmetric (fermions)

If 2 fermions are in the same state ($\alpha = \beta$) their wave function is 0 ! This problem does not exist for bosons which can occupy the same state (ex. supra- conductors).

This can be generalized for a larger system of particles.

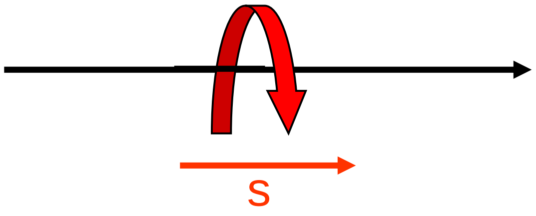
Helicity

- Particle of spin $\vec{S}$
- Axis orientation in the momentum direction $\vec{n}$
- Helicity :

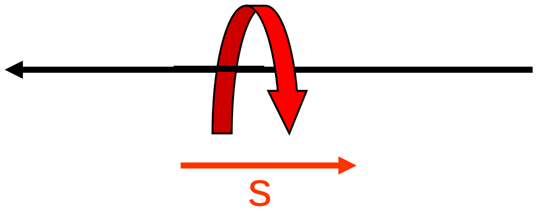
$$\Lambda = \vec{n} \cdot \vec{S} \quad \text{with} \quad \vec{n} = \frac{\vec{p}}{\left|\vec{p}\right|} \qquad \Lambda = \vec{n} \cdot \vec{J} \quad \text{because} \quad \vec{p} \cdot \vec{L} = \vec{p} \cdot \left(\vec{r} \wedge \vec{p}\right) = \vec{0}$$

- Eigenvalues $-s \leq \lambda \leq s$ $\qquad 2s+1$ values

- if mass=0 $\qquad$ only 2 eigenvalues : $\pm s$



Right-handed particle



Left-handed particle

The helicity is invariant under rotation (scalar product of 2 vectors).

APPENDIX III : two body space phase

$$d\Gamma = \frac{1}{2m_a}\sum_{\text{int}}\left|\langle f\,|\,T\,|\,i\rangle\right|^2 (2\pi)^4\,\delta^4\left(Q_f - Q_i\right)\prod_{k=1}^{2}\frac{d^3p_k}{(2\pi)^3\,2E_k}$$

$Q_i = q_a + q_b$ quadrivecteur energie impulsion de l'etat intial $|i\rangle$

$Q_i = \sum_{k=1}^{2} q_k$ quadrivecteur energie impulsion de l'etat initial $|f\rangle$

$\vec{p}_k, E_k$: impulsion, energie de la k^{ieme} particule finale

$$d\psi = \int_4 (2\pi)^4\,\delta^4\left(Q - q_1 - q_2\right)\prod_{k=1}^{2}\frac{d^3p_k}{(2\pi)^3\,2E_k}$$

On intègre sur 4 variables à choisir parmi les 6 impulsions afin de faire disparaître la fonction δ qui représente la conservation de l'énergie-impulsion

On se place dans le référentiel du centre de masse : $Q = \left(\sqrt{s}, \vec{0}\right)$

$$d\psi = \int_4 (2\pi)^4\,\delta\left(\sqrt{s} - E_1 - E_2\right)\delta^3\left(\vec{p}_1 + \vec{p}_2\right)\frac{d^3p_1}{(2\pi)^3\,2E_1}\,\frac{d^3p_2}{(2\pi)^3\,2E_2}$$

Après intégration sur les 3 composantes de p_2

$$d\psi = \frac{1}{4(2\pi)^2}\int_1 \delta\left(\sqrt{s} - E_1 - \sqrt{p_1^2 + m_2^2}\right)\frac{d^3p_1}{E_1\sqrt{p_1^2 + m_2^2}}$$

On choisit maintenant d'intégrer sur E_1 et on utilise: $p_1^2 = E_1^2 - m_1^2 \qquad d^3p_1 = p_1^2\,dp_1\,d\Omega_1 = p_1 E_1\,dE_1\,d\Omega_1$

$$d\psi = \frac{1}{4(2\pi)^2}\int_1 \delta\left(\sqrt{s} - E_1 - \sqrt{E_1^2 - m_1^2 + m_2^2}\right)\frac{\sqrt{E_1^2 - m_1^2}}{\sqrt{E_1^2 - m_1^2 + m_2^2}}dE_1 d\Omega_1$$

Cette intégrale est de la forme $\int g(x)\delta\left(f(x)\right)dx = \frac{g(x_0)}{\left|f'(x_0)\right|}$ avec x_0 tel que $f(x_0) = 0$

avec : $f(E_1) = E_1 + \sqrt{E_1^2 - m_1^2 + m_2^2} - \sqrt{s} \quad \Rightarrow f'(E_1) = 1 + \frac{E_1}{\sqrt{E_1^2 - m_1^2 + m_2^2}}$

$$g(E_1) = \frac{\sqrt{E_1^2 - m_1^2}}{\sqrt{E_1^2 - m_1^2 + m_2^2}}$$

$$\frac{g(E_1)}{f'(E_1)} = \frac{\sqrt{E_1^2 - m_1^2}}{\sqrt{E_1^2 - m_1^2 + m_2^2}} \cdot \frac{\sqrt{E_1^2 - m_1^2 + m_2^2}}{\sqrt{E_1^2 - m_1^2 + m_2^2} + E_1} = \frac{\sqrt{E_1^2 - m_1^2}}{\sqrt{E_1^2 - m_1^2 + m_2^2} + E_1}$$

La fonction $f(E_1)$ s'annule pour la valeur E_{10} telle que $E_{10} = \frac{1}{2\sqrt{s}}\left(s + m_1^2 - m_2^2\right)$

E_{10} est la valeur qui correspond à l'énergie de la particule 1 dans le centre de masse

$$\frac{g(E_{10})}{f'(E_{10})} = \frac{\sqrt{E_{10}^2 - m_1^2}}{\sqrt{E_{10}^2 - m_1^2 + m_2^2} + E_{10}} = \frac{p_{10}}{\sqrt{s}}$$

$$\boxed{d\psi = \frac{1}{4(2\pi)^2}\frac{p_{10}}{\sqrt{s}}d\Omega_1 = \frac{1}{16\pi^2}\frac{p^*}{\sqrt{s}}d\Omega_1}$$

Integrated luminosity

- Product of the luminosity of a characteristic time (1 year .. , experiment lifetime …)

$$L_{\text{int}} = \int L \; dt \qquad cm^{-2} \qquad \text{or barn}^{-1}\left(\text{b}^{-1}\right)$$

- PEP-2 example $L = 3\ 10^{33}\ \text{cm}^{-2}\ \text{s}^{-1}$ 1 year (~ 10^{7} seconds)
- $L_{\text{int}} = 3\ 10^{40}\ \text{cm}^{-2} = 30\ \text{fb}^{-1}$
- $N = \sigma\ L_{\text{int}}$
- production cross section of the l'Υ(4s) : ~1.1 nb

$\Rightarrow$ 33 10^{6} Υ(4s) produced by year by the PEP-2 machine

$1\ \text{b} = 10^{-24}\ \text{cm}^{2}$

$1\text{fb} = 10^{-15}\ \text{b}$

$1\text{fb}^{-1} = 10^{39}\ \text{cm}^{-2}$

L_{int} takes into account the machine operation : convenient !